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Journal
of
The Franklin Institute
Devoted to Science and the Mechanic Arts

Vol. 204 AUGUST, 1927 No. 2

POWER IN CALIFORNIA.*

BY

A. H. MARKWART.

Vice-president, Pacific Gas and Electric Company.

THE subject which has been assigned to me is "Power in California." Any discussion of power as it is universally distributed through the medium of electricity by an industry that is typically American in its constant expansion, initiative and wholesome inquiry into new inventions, new theories and new applications of science, is singularly appropriate before an Institute, rich in fine American tradition, organized for the very purpose of extending knowledge in "Science and Mechanic Arts," and in its activities, closely bound up with the development of electricity. The Franklin Institute is an exponent of that "Fifth Estate" or small "company of thinkers, workers, expounders and practitioners upon which the world is absolutely dependent for the preservation and advancement of that organized knowledge which we call science," [1] to which your attention was directed by Arthur D. Little on the occasion of the Centenary Celebration of the Founding of the Institute. This estate contributed the trained intelligence, vision and scientific imagination which has caused electricity to become the means of lifting the burden from labor and one of the inspirations for the practical

* Presented at the Stated Meeting held Wednesday, April 21, 1927.

[1] "The Fifth Estate," Arthur D. Little, Chem.D.

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idealism which to-day is one of the dominant qualities of the spirit of America. And, the professional spirit of service animating this social group has permeated the electric utilities and finds complete expression in their public policies. Indeed, were Francis Bacon here to-day he would indubitably select only those who are members of this "Fifth Estate" to sit in the councils of state of his New Atlantis.

Like California, your state is far advanced in power development. Therefore, to avoid carrying coals to Newcastle, I will confine my remarks to my state, as far as this is possible in the telling of the story. I shall endeavor to avoid a provincial attitude often attributed to Californians and forthwith allay any fears which might arise that "Young Lochinvar is come out of the West."

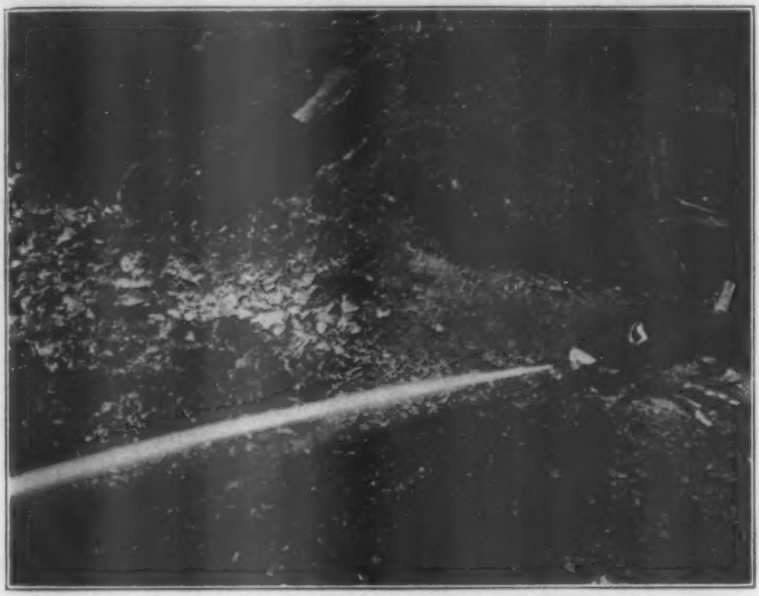
My purpose is to treat historically the several factors which brought California its general power system, to touch upon certain economic and social aspects of power and its function in state expansion, to express an opinion on the rate structure and on the attitude of the electric utilities toward the consuming public, to compare the electric service development in California and the nation, and to indicate some of the problems before the engineer in a state where hydro power is available and steam power is necessary, to review the fundamentals of power cost, and to present an economic study in this latter connection.

HISTORICAL.

The old saying, "Coming events cast their shadows before," is peculiarly applicable in the story of hydro-electric power in California. Few, even of those who live in this state, realize that the discovery of gold, an epochal event, while of direct political and economic importance at the time and subsequently, was but the precursor of even greater things. Gold mining has always had a fascination for Californians, and according to tradition, the Spanish explorers themselves, inspired by the romance, "Las Sergas de Esplandian," by Montalvo, published in Spain in 1510, considered that the "California" of this romance, where gold and precious stones were found in abundance, was that unknown country north of Mexico.

Curiously enough, the hydraulic "monitor," operated with

water from the early ditch systems, which mined the auriferous deposits, and the hurdy-gurdy, logically following, which permitted deep mining on the original veins of the Sierra Nevada, foreshowed the general use of the power of falling water. They were harbingers of the widespread power supply which has contributed so largely in bringing California into the industrial field. While the portent and effect of events on the electric power indus-



Monitor being used in hydraulic sluicing similar to hydraulic gold mining methods in the early days in California.

try are more significant than dates, the story will nevertheless be enriched if a brief chronology of some years of critical occurrence is given.

John Marshall discovered free gold in 1848 at Coloma on the South Fork of the American River.

In 1852 Edward E. Mattison, a Connecticut Yankee, with a view to economizing labor, used a stream of water under pressure with his invention of the monitor.

Gold-quartz veins were discovered in 1851 and the use of the hurdy-gurdy, the offspring of the monitor, followed in due course. Starting about 1866, modifications of the hurdy-gurdy

were introduced for the purpose of improving the efficiency or increasing the output.

The first noteworthy experiments on hurdy-gurdy wheels were conducted by Hamilton Smith, Jr., along in 1875.

Although J. B. Francis, of Lowell, Mass., invented the Francis type of reaction water turbine for low and medium heads in 1849, this type of water wheel was not employed in water-power practice in California until many years later on account of the early practical development there of the impulse wheel where the conditions for the development of water power made such a wheel necessary. It was better adapted for use under the high heads which had been produced for the purpose of hydraulicking the gold-bearing gravels, and made available more readily and immediately the motive power that was essential to the conduct of quartz mining.

The use of water power was greatly facilitated through the development, by Lester Pelton in 1880, of the Pelton type of bucket for use on improved impulse wheels. This development made possible efficiencies that were not obtainable with the crude hurdy-gurdy, and was the first step in placing the impulse wheel in the position it now holds as an efficient prime mover.

Coupled chronologically with the mechanical improvement of the water wheel, as a prime mover, were two other factors which created a triumverate that was to render feasible the extensive development of hydro-electric power in California.

The second of these elements was the establishment in 1879 of a small central station in San Francisco, furnishing a limited steam-electric service, which germinated the demand for electric energy and its compounding growth which we see to-day.

The third of these agencies was the demonstration by Italian engineers, in 1886, that electric current could be sent through a wire a distance of seventeen miles to Rome, from their steam plant at Cerchi. Thus came into being the means of connecting the water power lying potentially in the mountain reservoirs and along ditches of the miners, with the power load in the cities.

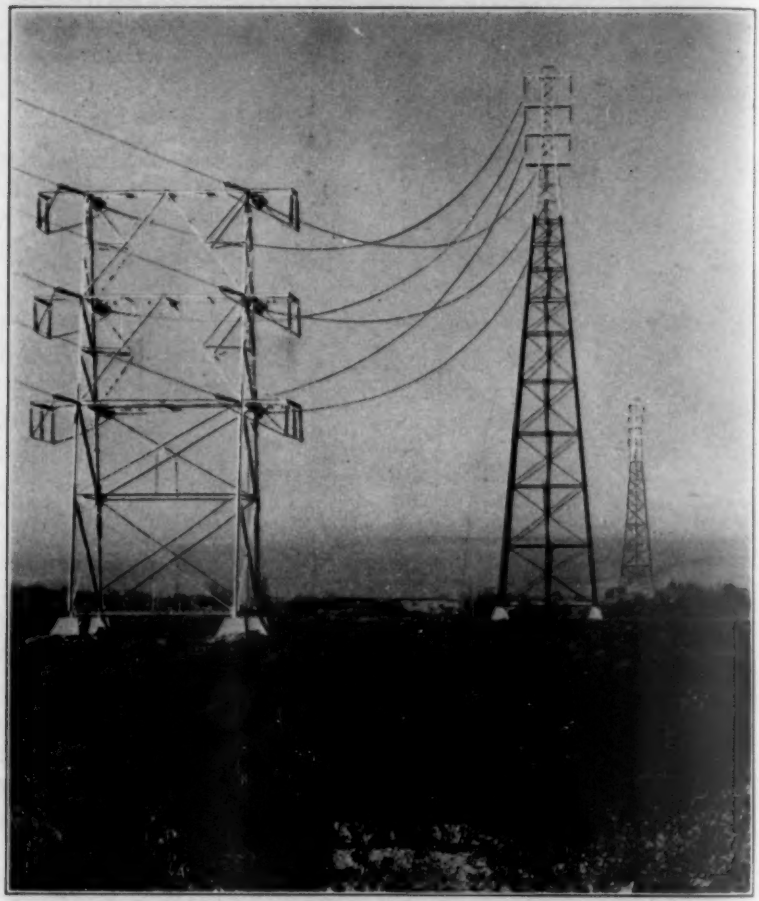
The coördination of these factors—a water power to serve a distant demand through the agency of a transmission—was secured, for the first time in the world, in Oregon in 1889, when

A 220-kv. transmission line in the snow country.

UNIV OF MIC

a hydro plant at Oregon City transmitted single-phase power at 4 kv., a distance of thirteen miles to Portland.

This was followed two years later by a 10-kv., single-phase transmission, 28¾ miles in length, from the Pomona Plant to



Sacramento River crossing of the 220-kv. line of the Pacific Gas and Electric Company. The centre tower is 459 feet high, believed to be the highest transmission line tower in the world.

San Bernardino. This was the first hydro-electric development in California.

The next decade saw a steady increase in transmission voltages and distances both in northern and southern California, together with the first 3-phase installation of any considerable size, the 3000-kw. plant at Folsom on the American River.

Of great importance in this progression was the first 60-kv. transmission accomplished by the Bay Counties Power Company in 1901 between Colgate and Oakland, a distance of 142 miles, in that its success inspired similar ventures at higher voltages, throughout the country, notably in California, Michigan and Colorado. In 1909 the Great Western Power Company of California completed its 154-mile, 110-kv. line from Las Plumas

The famous Kennedy Gold Mine in California, a modern electrically driven mine.

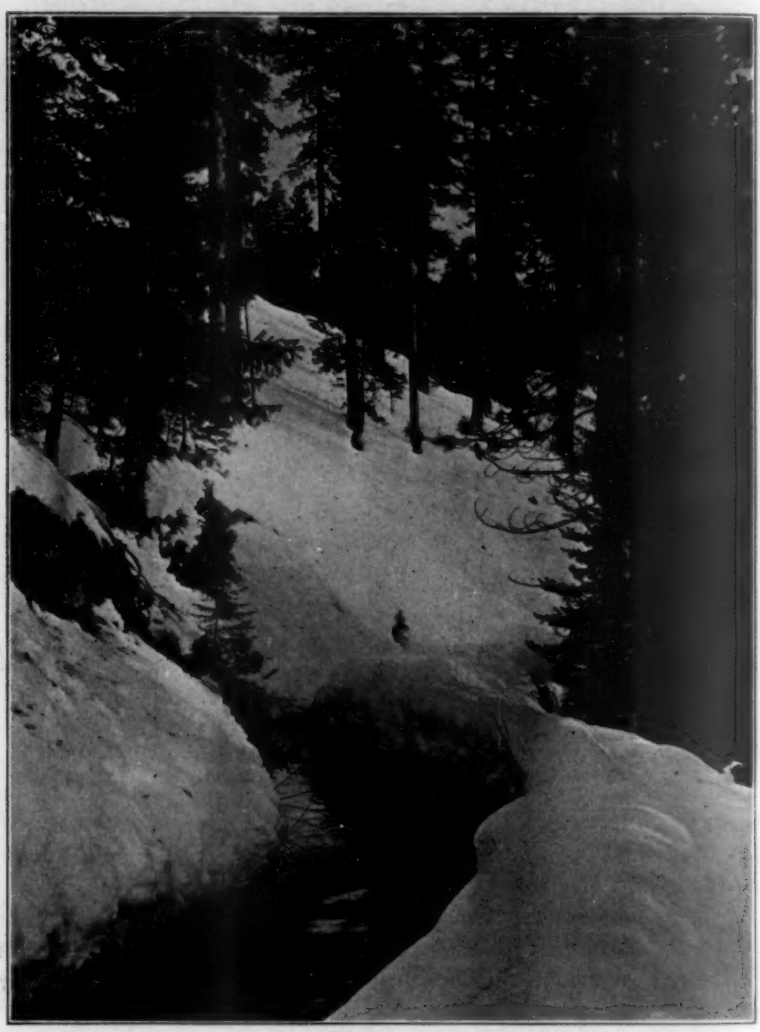
to the San Francisco Bay region. By 1913 a 150-kv. system of 241 miles had been constructed by the Pacific Light and Power Corporation,[2] from its Big Creek Plant to Los Angeles.

Ten years passed before transmission voltage was raised from 150 kv. to 220 kv. This took place in both northern and southern California in 1923, the Southern California Edison Company accomplishing it by reconstructing its 150-kv. system for 220-kv. operation, and the Pacific Gas and Electric Company by building its Pit transmission line from the Pit River to the Vaca-Dixon Substation, 202 miles.

The first modern high-pressure, high-temperature, steam tur-

[2] Subsequently merged with the Southern California Edison Co.

bine plant installed on the Pacific Coast was completed in Los Angeles by the Southern California Edison Company in 1924,



Boardman Canal, an old mining ditch, converted to carrying water for electric power generation.

this installation being largely inspired by the deficiency in precipitation during the 1923–1924 season. This plant, consisting of two units of a rated capacity of 35,000 kv.-a. each, operates

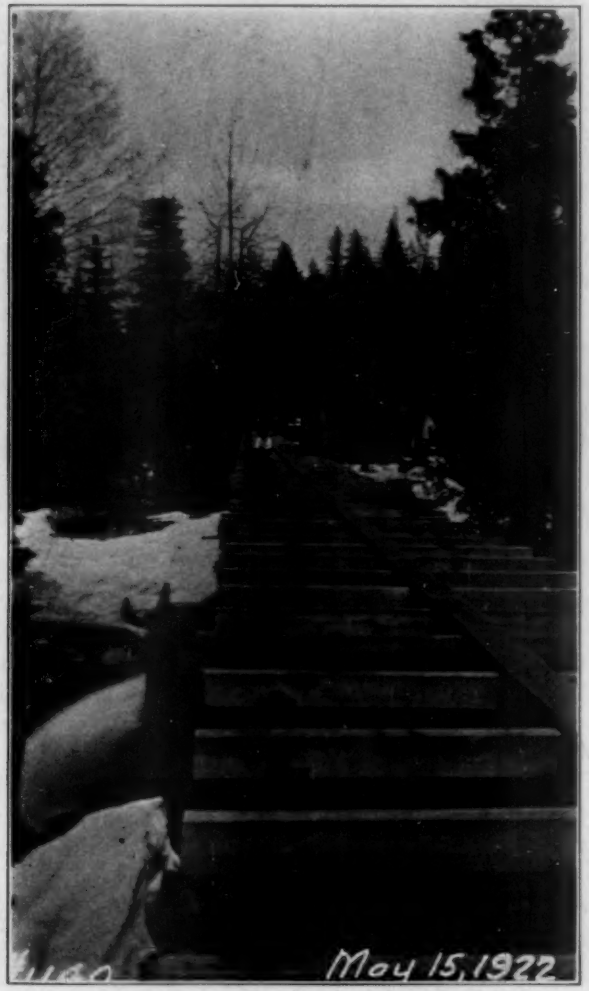
with about one-half the fuel consumption of the average Pacific Coast plant, the steam plant performance of plants then existing and still functioning being to the order of 30,000 B.t.u. per kilowatt-hour.

While gold mining cradled power development in northern California, it was not only mining and its water-power phases that started the regional power system off on a hydro-electric basis. Steam power for direct drive was also employed at the mines, and the centres of population had steam-electric power before the advent of hydro-electricity. It was only within the last few months that one of the most important of the Mother Lode mines,[3] the deepest vertical-shaft gold mine in the United States, which had clung to the use of steam power, discarded it in favor of electric service. In the early days steam plants, to burn wood fuel, were installed for the hoisting of ore and the running of stamp-mills. The fuel problem was a serious one, as tremendous quantities of cordwood had to be transported and stored during the summer months. Obviously there was no economy in fuel consumption. In the cities coal prices ranged from $4 to $11 per ton, depending upon the point of delivery, whether at tidewater or inland, and fuel oil had just come in. Fuel economy, judged by present standards, was very low, being to the order of 100,000 to 50,000 B.t.u. per kilowatt-hour. The service of the steam plants which were in existence in every principal city of the state by 1886 was, accordingly, for economic reasons, confined to the narrow limits of the city distribution systems. Under these power conditions hydro-electric power was a natural result.

During the decade of 1890 to 1900 hydro-electric development commenced almost simultaneously in northern and southern California, and both sections of the state have steadily progressed with the building of their general power systems. It was perhaps the presence of the miners' ditches on the Sierra streams from the Yuba to the Merced, and the primitive steam plant practice in the cities, and at mines where steam power was employed, which prompted hydro-electric development in northern California. On the other hand, the desire to furnish power to manufacturing industries at a price equivalent to that in localities where

[3] Kennedy Mine at Jackson, Amador County, which has a vertical shaft 4614 feet deep to the bottom of the sump. The deepest level is the "4500."

coal was $4 per ton, brought it about in southern California. A system of miners' ditches was the nucleus of the Pacific Gas and Electric Company in northern California, and $11 coal at Red-



Flumes on the South Yuba Canal of the Pacific Gas and Electric Company. Flumes like this were first used in California to carry water for hydraulic mining.

lands in 1892 accounts for the Southern California Edison Company in southern California. The Great Western Power Company in northern California had an early but unconscious beginning

when the 2½-mile tunnel was cut across the Big Bend of the North Fork of the Feather River by the miners to permit the

Nevada power house, one of the early plants of the Pacific Gas and Electric Company, now abandoned. Constructed in 1896. Capacity, 1260 kv.-a. Heads, 206 and 785 feet.

working of the free gold which was suspected to lie in the river bed. The tunnel is now used for the generation of electric power.

Some of the early-day plants still exist, notably the Folsom

Plant of the Pacific Gas and Electric Company which was built to supply the City of Sacramento, and the Mill Creek No. 1 Plant of the Southern California Edison Company to supply the City of Redlands. Each of these plants was the result of the then extremely poor steam plant performance and relatively high-priced fuel. However, a number of other plants in northern California were built to displace the water or steam power at the mines, and soon after the mines had received electric power, the nearby towns, which had their birth in early mining days, and the agricultural areas surrounding them, were connected to the promising young hydro-electric systems which had sprung up here and there by 1898.

Along in 1899 the Colgate Plant on the North Fork of the Yuba came into the picture. It was an ambitious undertaking, brought about by the success obtained with the smaller Nevada Plant on the South Fork of the Yuba River, and the Yuba Plant which took water from an old mining ditch that had its diversion on the North Fork of the same river.

In 1898, because of a shortage of water at Folsom, the idea of connecting two remote power supplies was conceived. In the following year, Sacramento, which had a steam plant, received power from the Colgate-Folsom interconnected power system. This proved later to be an occurrence of tremendous importance, for such a tying together of hydro-electric plants and a steam plant to supply light to cities and to serve industry and agriculture with power, established the principles:

First: That advantage could be taken of the diversity which exists in stream flow;

Second: That a steam reserve furnished the desired continuity to hydro-electric service; and

Third: That pooling of a regional demand, which varied daily, seasonally, geographically and with character of energy usage, resulted in economy, owing to the greater number of hours which a given plant capacity could be effectively operated.

Following the notable transmission feat of the Bay Counties Power Company, its Colgate-Oakland Line, the system of the Standard Electric Company was brought into service in 1903. As the name of the forerunner company implies, the rich load centre about San Francisco Bay was the real objective of both. The Electra Plant of the latter company, in the mining region on the

OREGON
INSTALLED PLANT CAPACITY
MAP
Showing
TRANSMISSION SYSTEMS
OF
CALIFORNIA
Scale in Miles
MEXICO

Mokelumne River, was connected by a 60-kv. transmission line to San Francisco Bay region by way of Stockton and San Jose, and to the Colgate Plant by means of a branch line to Oakland. Aside from the importance of these lines as forward steps in transmission practice, the interconnection of these companies at Oakland stimulated the interconnection idea previously developed in a smaller way at Sacramento.

These physically connected plants furnished the prototype of the great consolidated hydro-electric, steam-supported power systems which now give California its universal power supply, and this early acquisition of diversified markets for the infant central station industry ushered in the high system load factors which prevail in California to-day.

It was but natural that consolidation of companies took place in due course, the Pacific Gas and Electric Company of northern California being a notable example. It is not the intention to review the history of the growth of the power companies of California. Suffice it to say that southern California had a similar power system growth, the Southern California Edison Company, serving Los Angeles and cities clustering thereabout, being the outcome of the pioneer plants on Mill Creek.

The 60-kv. transmission which generally prevailed at the time the original power pool was started, could be tapped en route because this voltage permitted substation costs which bore a proper economic relation to the size of the loads to be carried. The significance of this probably was not realized at the time, but in a measure this economically suitable voltage made it possible for industry to locate in rural communities, and is responsible to some extent for their growth.

It safely may be asserted that if, along in 1895, the art of steam-electric power generation had been advanced thirty years, steam plants would have prevailed and a power supply for rural territory would have been long delayed. So, it was fortunate that California engaged in the hydro-electric enterprise, that the power sources were so situated that the rich interior valleys had to be traversed with transmission lines to connect these sources with the major markets on the coast, and that the high voltage possible for transmission with the then state of the art was as low as 60 kv.

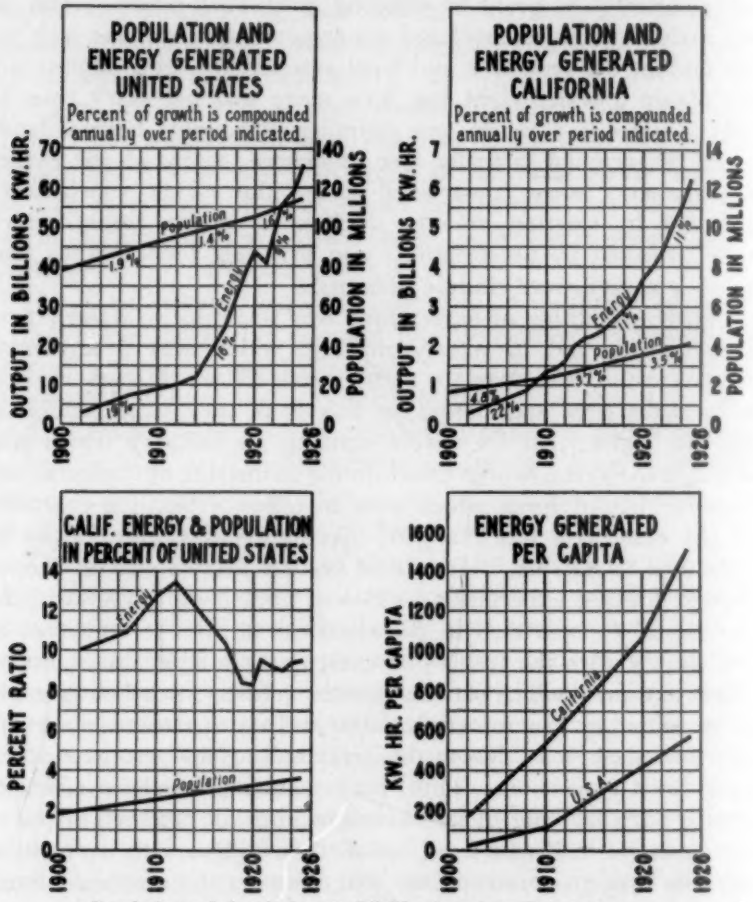
ECONOMIC PHASES.

The general utilization of water power in California was possible when the cities had built up a load on steam-electric plants and when it appeared to enterprising groups that there was present an opportunity to profit by shutting down such plants. And, in the early days an undeveloped water-power project was had for the taking, the economic and legal restrictions being almost nil. To obtain a water right but little more was necessary than to affix a notice to a nearby tree claiming such right; Federal lands could be occupied virtually free of charge; many of the earlier developments were constructed under the rights which were attached to existing mining ditches or under rights which were acquired by proscription; and privately owned lands that were necessary were cheaply obtained.

Such a condition of *laissez faire* was favorable to water-power development, and to money making, which was a legitimate reward to the promoters for constructive effort. However, these early promoters, speculators in power as they were, doubtless did not realize that they were starting an industry which was destined to exert a benign effect on the industrial, agricultural and domestic life of times which were to follow. But, the character of the enterprise has changed; speculation has given away to industrial leadership, and central station service has developed beyond fondest expectations. Power lines have multiplied, and the capital concentrated in the electrical utilities is employed at low wages, with the result that energy is available almost everywhere at a price which permits general industry, small or large, to locate wherever raw materials, labor and transportation are available and allow it to devote the great bulk of its resources to its own direct expansion. Thus, manufacturing has been added to mining; dry farming of vast acreages in grain has been displaced by intensive cultivation of smaller holdings with irrigation; payrolls have grown in number and amount; and economic diversity for the state has been the by and large result.

The public utilities of California have a keen appreciation for the idea that they have duties as well as privileges. While they have thrived under private ownership, there has been due regard for the interests of the consuming public. The control, which is largely Californian, in its desire to create conditions favorable to general industry and pleasant living in a state not yet fully

developed, may have something to do with this. Or, a greater degree of public regulation and socialization may tend to prevail here than elsewhere. Perhaps this causes the industry to proceed rationally along the lines of least resistance, recognizing that suc-



Population and electric output of California and the United States.

cess in the long run will be had by agreeing with, rather than opposing, public sentiment, as far as this is possible without destroying the individualism which has placed the industry where it is to-day and which must be retained to furnish the personal and vital interest which is essential if the industry is not to be aquiescent. Whether the actuating motive of the industry, which

flowered from a desire within, is called altruism or enlightened selfishness is of no moment, for "that which we call a rose, by any other name would smell as sweet." All that the electrical industry of California desires is a financial return sufficient to maintain itself in that economic position which will permit an expansion sufficient to meet the needs of the growing state. The industry is fully aware that the rate at which it can sell energy will go down as the use per average consumer goes up.

The rate structure in California, like its interconnected power system, is an evolved framework. The business of rendering electric service is monopolistic in character. Under regulation the industry is allowed to collect its expense and to receive a return; expense including the charges for operation and maintenance, annual depreciation, taxes, insurance and management outlay; and return, theoretically not to exceed 8 per cent. on the value of the physical property, used and useful, and on the cost of franchises, water rights and other intangibles, to cover interest on borrowed capital and on reserves invested in utility property, bond discount amortization and expense, sinking fund requirements, dividends and amounts carried to surplus. At least this is the theory of the business, and there is nothing available for profit, as this is commonly understood. The rate schedules are strongly influenced by experience, but they possess an element of the scientific to the extent of having the different classes of consumers pay the cost of the service as far as this is practicable by spreading the total annual costs of the utilities in the form of rates.

The principal social elements enjoying the service of the public utilities are the domestic consumer, industry including mining and transportation, and agriculture. Domestic consumers are divided into two general classes, those in the cities, and those in the rural communities.

All consumers enjoy certain economic benefits resulting from regional systems, for the cost of service to them would have been greater than it is, had the systems developed on a community basis. In the common good, the large cities have contributed a dense business, whereas the rural territory, upon which the cities depend, by taking service, has promoted higher load factors and greater diversity. It is sometimes claimed that in the spread of rates the country has enjoyed the greater benefit, but if this be true it must be remembered that a prosperous back country is

essential to a city's progress. There is therefore no disparity in the contribution of the outside areas toward the creation of a situation which is favorable to universally low service rates.

All rates neglect consideration of the geographical position of the consumer. The industrial consumer, located adjacent to the power source, receives no less rate than one hundreds of miles away. Nevertheless, all industrial rates take into account the

Lake Spaulding storage reservoir of the Pacific Gas and Electric Company. Capacity, 75,000 acre-feet.

wholesale character of their usage, and these rates are influenced to some extent by the cost of power from other sources. Industry benefits thereby, but this spread is likewise in the common interest, in that industry has not had to carry too great a burden, thus stimulating its expansion in a state not yet industrially complete.

On the whole, the first thought is the good of the Commonwealth, and there has been no direct discrimination in favor of any class of consumer. For example, the small consumer, because he is in a political majority, is not tendered an unbalanced bid for his business, so, while not wholly scientific, the rate structure at least is not political. If it could be characterized in any way,

it could be said to rest on an equitable rather than scientific base, meaning that it is of a character which tends to create and preserve material prosperity for the state as a whole. Further development of industry will result in higher regional load factors, and lower prices for energy to all, including domestic consumers. All rates for service are less than its value to the consumer, and they are such as to produce an average annual selling price per unit of energy considerably less than that which obtains on the average for the country as a whole. What the traffic will bear is never considered, in fact, one is not going too far in saying that the electric utilities of California have the fundamentals of public service clearly in mind and are not actuated solely by gainful motives. What is sought from the regulatory body is a price for service which will be reasonable to the consumer after a consideration of all of the sound factors which go to make rates and that which will insure a net return just sufficient to attract the capital which the business needs to continue its expansion.

As indicative of the extent of energy used, the production of energy per capita per year is now 1600 kw.-hrs., over two and a half times that produced per capita in the nation, and California with 3.5 per cent. of the population produces 9.5 per cent. of all the energy. The growth in the production of energy is now at the rate of approximately 11 per cent., compounded annually, or over three times the rate of growth of population. In contrast to this, it is interesting to note that the rate of growth for the United States is now about 9 per cent., compounded annually, or five and a half times the rate of growth of population. The observation to be made regarding the relation existing between the energy output and population growth in the two cases is to the effect that the United States is working toward that degree of power saturation which obtains in California. Were the nation saturated to the extent that California is, the rate of growth of energy in the United States would be to the order of 5 per cent., compounded annually, instead of the 9 per cent. which obtains.

Among other things, the industry has succeeded in effecting the spread of electricity by coördinating the dense business of cities with the scattered business of country communities. With 62 per cent. of the farms of the state electrified, it is demonstrated that rural electrification is economically feasible. The extension

of electric facilities has encouraged decentralization of population, has synchronized the development of the city and the country, has furnished the widest basis for the development of industry, and has fostered industrial, agricultural and domestic life in sections of the state which were inactive but potential. As illustrating this last point, there is the gold mining by electric dredging of the auriferous gravels to depths as great as 86 feet, which industry has been successfully conducted since as early as 1901, after electric power had become available. The value of gold produced by dredging, from 1898 to the close of 1924, was over $140,000,000. Furthermore, electric power will bring about gravel mining to greater depths as soon as some genius overcomes the mechanical difficulties attached to the problem. When this is done, there will be a resumption of the mining of the deep gravel banks, the hydraulicking of which was prohibited by Federal enactment in 1884, on the grounds that the "slickins" or débris deposited in the rivers interfered with navigation, although the underlying complaint originated with the farmers in an endeavor to forestall the deposit of gravel on the agricultural lands in the valleys below the mining region. Approximately $300,000,000 worth of gold was mined by hydraulic methods during the period commencing with California's admission into the Union and ending in 1884, and it is estimated that over $100,000,000 more is susceptible of economic extraction from the gravel banks of the Sierra between Lassen County on the north and Madera County on the south.

Electric power made the Portland cement industry in California. In the early days the Pacific Coast imported all its cement. San Francisco was an importer as far back as 1864, in which year she received 13,000 barrels. In 1884 this had been extended to 150,000 barrels. The first plant to make true Portland cement on the Pacific Coast was established in Oregon City, Ore., in 1884. The mill was abandoned about 1890, because the stone measures were exhausted. The first California plant began operations in 1898 with central station hydro power at Colton, about fifty miles east of Los Angeles, with a capacity of 500 barrels per day. At the end of 1924 there were seventeen plants in operation on the Pacific Coast, capitalized at about $43,000,000 with a combined capacity of 59,000 barrels daily. The output for California alone in 1925 was 36,200 barrels per day. The

energy consumption in California is from 15 to 17 kw.-hrs. per barrel. The cement industry may be said to have commenced its active expansion in 1901 about the time that hydro-electric power was an accomplished fact, and since that time the industry in California and on the Pacific Coast as a whole has had a remarkable growth.

Another example of electricity as an aid in potential territory was the furnishing of power to develop for rice growing, lands which were adaptable to no other culture.

Not including the fuel energy for steam for process purposes, space heating and cooking, nor for the steam railroads, the average energy requirement of the State of California in 1926 was approximately 834,000 kw. All of this energy was supplied electrically except some 45,000 kw., or approximately 5 per cent., so the state appears to be 95 per cent. saturated with electric power. The connected electrical load was 6,000,000 hp., or approximately 2.25 times the installed generating capacity. Future additional business of the central stations will include power to meet natural and stimulated growth for character of uses served in territory now covered, that for present usage in territory not now covered, that required for railway electrification and that required to serve created industry. Regarding the last-mentioned purpose, because of the degree of saturation which prevails, it is becoming increasingly important to consider those industries which are potential users of power. To emphasize this feature, the utility companies, with a view to coördination in industry, coöperate closely with the business and industrial agencies of the state in studying the possibilities for creating new industry to further the state's industrial expansion and to afford employment to the many coming here to reside; for developing wider, and enriching existing markets; for increasing the consumption in California of home-produced goods, and for increasing the fabrication of finished products within the state from raw materials produced in, tributary to, or passing through the state.

As an example, raw silk to the value of $77,000,000 entered the port of San Francisco in 1925 on its way from the Orient to eastern manufacturers. Sericulture in California is in an experimental stage with indications favorable and the industry of silk spinning and weaving is of negligible proportions, although the Pacific Coast is now importing from the East manufactured

silk products to the order of $100,000,000 per annum. Cotton growing in California is likewise in its infancy, there also being a relatively small volume of finished cotton goods manufactured here, although cotton products to the value of $70,000,000 are required annually for use in this state. California has a large wool clip, which, together with Pacific Coast imports from Australia, New Zealand and the Orient, indicates a huge raw supply on the Pacific Coast. There is very little Pacific Coast production of finished woolen goods, and California has but one woolen mill. From the examples cited it is evident that there are great volumes of raw materials close at hand, the conversion of which into finished goods elsewhere does not contribute to the wealth of California. As a matter of fact, the climatic, soil and other conditions in California are favorable to integrated textile industry in silk, cotton and wool from the production of the raw materials to the weaving of the fabrics. Power will be necessary for future industry in these respects, and further, there are the electrochemical, electrometallurgical and heavy transportation fields which have not been entered; the gold is yet to be extracted from the deep gravels; and the mining of low-grade coal from measures now being explored is not an impossibility. The ideal living and working conditions which obtain in California, together with the contribution of the electric power industry in the form of an abundance of cheap power, assure the expansion of California and its continued economic progress.

To some extent things have changed since the construction of the first hydro-electric plants, through the introduction of features which are developing an upward trend in capital costs. Undeveloped water resources are now considered as possessing great value by individuals from whom it is necessary to acquire lands or rights to permit development and such owners consider themselves entitled to a proportion of the unearned increment of value which they believe is created by the constructive work of the power companies. There are many legal restrictions relating to the use of public lands and title in fee of such lands cannot be obtained; state and Federal licenses for use of streams for power generation are exacting; actions at law involving water rights, wasteful of time and capital, are frequent; compensation for damage, real and imaginary, is high; rights of way are becoming more expensive as the country develops; nuisance values are readily created;

invisible costs are mounting rapidly; and with few exceptions it is now the distant, less accessible, larger and more costly projects that remain to be considered for development.

Moreover, with the distance that power is now being transmitted the superior economy of the highest practical voltages is apparent, so to keep transmission unit costs within the range of economy, higher voltages are the necessary resort. High voltages in turn have brought wider rights of way, higher towers, more line insulation, corona suppression, synchronous condensers for voltage control and expensive circuit breakers and costly terminals.

The western engineer must and is introducing these social and technical factors into the economic equation. Also his problem, where long-distance, high-voltage transmission lines are involved, is one which has in it such phases as balancing the cost of spare transmission capacity and costly sectionalizing and terminal switching, against cost of steam reserves, and the cost of obtaining peak capacity from steam installations as compared with the cost of extra capacity in hydro plants and transmission, bearing in mind that steam plant capacity acting as a reserve or to comb peaks is also available, with the burning of additional fuel, to meet exceptional demands for energy occasioned by more rapid growth than was anticipated, and to make up the seasonal and annual deficiencies of the hydro system.

California has, in effect, but two seasons, the wet and the dry, the winter or rainy season usually commencing in October and ending in April. After the first of May precipitation rarely occurs, except summer showers in the high mountains. The precipitation is in the form of snow at the higher elevations of the Sierra Nevada and Cascade Mountain Ranges, and falls as rain in their lower foothill reaches. The runoff from the rains during the wet season and the melting snow in the dry season feed the streams of the western slope of the Sierra and the Cascades, upon which most of the power developments are situated. Elsewhere in the state the precipitation is always rain except in the northerly section of the Coast Range which is favored with both snow and rain. The precipitation reduced to water varies from three inches on a dry year in arid sections of the state to 113 inches on a wet year in "watered" regions.

Under these variable meteorological conditions most streams are very erratic in their flow. From flood flows in the spring

measured in thousands of cubic feet per second the stream yield in the late summer and autumn is often reduced to a negligible amount. As a result storage is essential on all but perhaps two streams in the entire state, the Pit and Klamath Rivers, which drain a section of the Cascades, to regulate the flash flows in order to effect monthly stream discharge more agreeable to the monthly demand for kilowatt-hours. Stored waters are released during the short water season extending from three to six months, depending upon the geological character of the watershed and the amount of annual precipitation. Storage sites of sufficient capacity to equate annual flows are not always present. It follows that power plants on such streams become stream flow plants with monthly outputs of energy varying from essentially nothing to a maximum as limited by the plant capacity. The difference between the actual monthly output and the share of the monthly load demand carried by such stream flow plants, usually must be met by burning fuel in steam plants.

Were meeting the seasonal deficiencies in hydro energy by storage works or steam plants the only consideration, the problem of wise and comprehensive planning would be relatively simple. A complexity arises out of the fact that the yearly runoff of streams varies over a considerable range owing to the more or less periodic occurrence of dry years. California has always had a "gamble with the weather." In no lesser degree than did the cattle man and grain farmer, does the power company realize the significance of this gamble, for such it is. The drought of 1864, which not only destroyed grain but hundreds of thousands of cattle,[4] was one of the factors which contributed to the breaking up of the enormous land holdings measured by tens of thousands of acres which were characteristic of California agriculture up to about 1870, and to the introduction of irrigation with the production of diversified crops.

Practically all water-power plants in California are subject to the disadvantage which attaches to the variability of precipitation and corresponding runoff. This variability greatly complicates the successful development of hydro power, since either water must be wasted in large quantities on wet years or steps must be taken to supplement the deficient flow on dry years. This

[4] Sixty thousand head of cattle were sold in Santa Barbara that year for 37½ cents a head.

weather risk, while of common knowledge to Californians, was not gaged with respect to its effect on power production until the last ten years, during which time the state has experienced a number of dry years and strangely enough has had the greatest expansion in its electrical power and light industry. Thus, it is only of late that the power companies in California, in spite of their having operated hydro plants of considerable size for about thirty years, with reliable runoff records for about twenty years of this time, have become aware of the magnitude of the deficiency in the output of hydro plants on dry years. This hydro deficiency is to the order of one-third in energy output, with perhaps a fourth of this in plant capability. Consequently it becomes important to consider the amount of supplementary power which is necessary to meet such deficiency in the future, if hydro plants are to be designed for normal flows, which is likely, and if there is to be no curtailment of service. This and related subjects were discussed by the writer in a paper presented at the Spring Meeting of the American Society of Mechanical Engineers of San Francisco, June, 1926.[5]

The fact that steam plants are necessary in the long run to supplement hydro power against failure of a generating or transmission unit and to make up the deficiencies in energy, both seasonal and annual, creates a situation in which it will often be possible to install less hydro capacity to care for growth of load than would be the case were steam plants not available. This idea goes to economy in power production costs, in that it is possible to comb off a portion of the system peak with the steam plant, supplying the relatively small number of annual fuel kilowatt-hours, and providing a higher load factor for the hydro, which is essential to its economy on account of the high capital costs and corresponding fixed charges which usually obtain for hydro projects. The amount of the peak that can be carried on steam to produce the best overall economy will depend upon the system load factor, the capital cost of hydro plants, including transmission, the cost of money, the cost of fuel, and the expenses of operation and maintenance, including depreciation. With this economic phase we shall deal in detail in due course.

Many of the older steam units installed in California are now

[5] A. H. MARKWART, "Aspects of Steam Power in Relation to a Hydro Supply."

substantially depreciated and ready for replacement. This condition, coupled with the fact that modern steam equipment produces nearly twice as much energy as the older plants do from a given amount of fuel, will have an important bearing on the manner in which additional hydro resources are to be developed. The new order of steam power costs will tend to cause a postponement of the development of all but the cheapest of the remaining hydro resources, and to accelerate the building of modern steam plants which will carry a greater proportion of system loads.

It is a curious commentary that California started off with steam plants, that it retained them to supplement the hydro, and that it is increasing the amount of steam power as the hydro sources are developed. Out of a total installed generating capacity of over 2,000,000 kv.-a. as of January 1, 1926, over 650,000 kv.-a. were of steam. While most of the power companies in California are considered hydro companies, the "handwriting is on the wall," and in time, perhaps in a generation, when the hydro sources are fully developed, fuel again will be the principal source of power, with hydro contributing as far as it can toward the sum total of energy requirements. In the interim, however, hydro power will be developed and steam will be employed to give it as favorable a position as possible. Obviously it is desired to generate as much electricity from water power as the conditions allow, before we make too great a demand upon a wasting and exhaustible asset. Admittedly, it is important in the broadest economic sense to consider posterity by continuing to make practical use of our water powers until they are completely utilized. As a rule economy will obtain for the present by the use of a reasonable proportion of steam power, particularly as economies in steam practice are improving.

If seasonal and annual hydro deficiencies did not occur, the installed generating capacity of a given hydro system would tend to be but slightly in excess of the anticipated annual peak. Furthermore, were it possible to have a power factor of the system such as would permit the generators to operate at unity power factor, a load factor that approximated 100 per cent., no spare capacity, and no losses, each kilovolt-ampere of installed generating capacity would become a kilowatt functioning 100 per cent. and power costs would approach a minimum. Such an

ideal situation is impossible, but it suggests an economic par and leads to the consideration of a number of factors when an economic test is to be applied in any individual situation, among which are total production, transmission and distribution capital, output, sales and revenue per kilovolt-ampere of installed

TABLE I.

Electric Service Development in California and United States; Central Stations, Including Municipal Plants, 1925.

STATISTICAL DATA

	UNITED STATES	CALIFORNIA
Aggregate Capitalization of the electric power and light industry Jan. 1, 1926	$7,500,000,000	$711,000,000
Total generating capacity Jan. 1, 1926	24,343,793 kva.	2,000,000 kva.
Total output	61,158,857,000 kw.hrs.	6,182,000,000 kw.hrs.
Total gross revenue*	$1,423,000,000	$90,221,000
Probable sales*	49,354,857,000 kw.hrs.	4,342,000,000 kw.hrs.

	DEDUCED FIGURES UNITED STATES	CALIFORNIA	% CALIF. ABOVE OR BELOW U.S.
Average Capital per kva of installed generating capacity	$308	$356	+16
Average sale price per kw.hr.	2.88¢	2.08¢	-28
Output per dollar of capitalization	8.15 kw.hr.	8.69 kw.hr.	+ 7
Output per kva of installed generating capacity	2512 kw.hr.	3091 kw.hr.	+23
Capacity factor**	28.7%	35.3%	+23
Sales per kva of installed generating capacity	2027 kw.hr.	2171 kw.hr.	+ 7
Gross revenue per kva of installed generating capacity	$58.5	$45.1	-23
Ratio of value of capitalization to gross revenue	5.27	7.88	+50

* Resale omitted
** In terms of kva

generating capacity, capacity factor, output per dollar of capital, and finally ratio of capital to gross revenue. Obviously in the interest of low-priced power it is necessary to keep the total system capital per kilovolt-ampere of installed generating capacity low and the kilowatt-hour sales high.

As affording a practical illustration of these considerations,

basic statistics for the years 1925 and 1926 have been assembled for California and the United States, and from these the several significant units and ratios have been derived. The statistics

TABLE II.

Electric Service Development in California and United States; Central Stations, Including Municipal Plants, 1926.

	United States.	California.	Per Cent. California above or below U. S.
Statistical Data.			
Aggregate capitalization of the electric power and light industry, January 1, 1927	$8,400,000,000	$817,000,000	
Total generating capacity, January 1, 1927	26,812,710 kv.-a.	2,088,800 kv.-a.	
Total output	68,732,000,000 kw.-hrs.	6,900,000,000 kw.-hrs.	
Total gross revenue*	$1,571,000,000	$100,150,000	
Sales*	55,468,000,000 kw.-hrs.	5,000,000,000 kw.-hrs.	
Deduced Figures.			
Average capital per kv.-a. of installed generating capacity	$313	$391	+ 25
Average sale price per kw.-hr.	2.83¢	2.00¢	− 29
Output per dollar of capitalization	8.18 kw.-hrs.	8.45 kw.-hrs.	+ 3
Output per kv.-a. of installed generating capacity	2563 kw.-hrs.	3303 kw.-hrs.	+ 29
Capacity factor †	29.2%	37.7%	+ 29
Sales per kv.-a. of installed generating capacity	2069 kw.-hrs.	2394 kw.-hrs.	+ 16
Gross revenue per kv.-a. of installed generating capacity	$58.60	$48.00	− 18
Ratio of capitalization to gross revenue	5.35	8.16	+ 53

* Resale omitted.
† In terms of kv.-a. Divide by assumed power factor to put in terms of kw.

for the United States are taken from the "Annual Progress—Statistical Number" of the *Electrical World*, January 1, 1927; those for California are gathered from local sources. The derived figures are believed to be sufficiently accurate to reflect the economic position of the industry in California and the United

States at the present time. The whole is tabulated for convenient comparison.

The economic position of the electric power industry as of 1925 and 1926 in California and the United States is statistically described in Tables I and II. The 1926 statistics are subject to correction as they are based upon ten months' operation. The deduced figures for 1925 are interpreted and compared in what follows.

The average capital in generation, transmission and distribution facilities per kilovolt-ampere of installed generating capacity was 16 per cent. higher in California because it has a base load hydro supply which requires more capital per unit of installed capacity than does steam power, which is the major supply for the country as a whole. California has roughly one-third of its capacity in steam plants, and two-thirds in hydro, whereas the capacity in the United States is in the reverse order. This disadvantage is more apparent than real and is more than balanced by the output per installed kilovolt-ampere of generating capacity and per dollar of capital. These units are 3091 and 8.69 for California, compared with 2512 and 8.15 for the United States. The enhanced position of California in these respects by 23 and 7 per cent. is the result of the higher load factors which prevail. Many of the eastern companies serve metropolitan areas and have load factors around 45 to 47 per cent., as contrasted with load factors of from 55 to 60 per cent., and at times as high as 65 per cent., enjoyed by the regional companies of California. This favorable load factor condition also accounts for the 23 per cent. higher plant or capacity factor which obtains here. Owing to greater losses by some 50 per cent. occurring in California, the advantage of greater output per unit of generating capacity, or higher capacity factor, is not carried along to the full extent into sales per kilovolt-ampere of installed generating capacity which were 2171 kw.-hrs. in California, or about 7 per cent. above the 2027 kw.-hrs. average for the United States.

The gross revenue of California was $45.10 for each $356 of invested capital, and in the United States $58.50 for each $308, which indicates a ratio of capital to gross of 7.88 and 5.27, respectively. To express this differently, California has an investment of $7.88, and the United States $5.27 to do $1 of annual gross business. Thus electric capital in California is turned over in a

little less than every eight years, whereas in the United States this is accomplished in a little more than five years. At first glance one might conclude that California had less to offer the investor, but such a conclusion would be unsound for, as a matter of fact, the California utilities, because they are on a hydro capital basis, tend to have lower operating ratios, that is, the tendency is to pay fixed charges instead of fuel bills. Considering depreciation as a fixed charge, the operating ratio of western hydro is a third less than that of eastern steam on the average.

On the other hand, the consumer will observe that he, too, is in a favored position, despite the fact that his utilities have a total capital investment per unit of installed generating capacity 16 per cent. greater than the average obtaining for the United States, since the gross revenue per unit of capacity is 23 per cent. less. This consumer view is not academic, because the average sale price per kilowatt-hour is about 2.1 cents for California as against nearly 2.9 cents for the United States. In brief, electricity is sold in the western state at an average price per kilowatt-hour which is 28 per cent. less than the average price for the country as a whole, notwithstanding the relatively higher capital investment.

Referring again to the matter of losses, obviously these are bound to be higher in California on account of greater distance of transmission. They will, however, vary from time to time as lines are loaded with the output of new hydro projects and with the amount of steam power supplied to meet deficiency and interruption in the hydro supply. Foundational capacity in transmission during a dry year would tend to lower line losses, whereas transmission lines loaded to capacity on a wet year would have the opposite effect. Losses will vary from one-quarter to one-third of the output, being 29.7 per cent. in 1925.

The production peak and annual load factor which obtained for some of the principal companies in California in 1926 are both interesting and illuminating. The Pacific Gas and Electric Company peaked at 407,000 kw. on a load factor of 60.50 per cent. The Southern California Edison Company had a peak load somewhat higher and a load factor somewhat lower, namely 422,700 kw. and 60.2 per cent. The Great Western Power Company and the San Joaquin Light and Power Company followed, respectively, with 135,250 kw. and 59.0 per cent., and 131,490 kw.

and 54.0 per cent. These two latter companies recently were interconnected by a line 100 miles long, which has a capacity of 70,000 kw. at 165 kv. and 125,000 kw. at 220 kv. One circuit is installed and is now operating at the lower voltage. The capacity of this interconnection is the largest thus far established between two companies in California. Although these are separate companies, under the control of one holding company, they are more or less jointly operated. Joint operation during the last six months of 1926 resulted in a combined system peak of 251,660 kw., whereas the non-simultaneous peaks of the two systems taken separately aggregated 266,740. This indicates a saving in capacity of slightly over 15,000 kw. and a combined system load factor to the order of 64 per cent., which is considerably higher than that of either system when operated separately. This greater usage of facilities clearly demonstrates one of the advantages of system interconnection. All of the companies mentioned above are doing a regional business and may be contrasted with the Los Angeles Gas and Electric Company and the Los Angeles Bureau of Power and Light. The former, a private company with a peak of 68,200 kw. and a load factor of 35.6 per cent., manufactures its power in steam plants. The latter, a municipal bureau, with a peak of 121,700 kw. and a load factor of 47.2 per cent., purchases the most of its power from the Southern California Edison Company. Both distribute power in the City of Los Angeles.

The favorable load factor situation in California suggests the economic importance to the industry of the physically consolidated regional company. Those that are not regional have low load factors, a condition generally true of non-regional companies in any section of the country. It is obvious that the best results are obtained when companies are closely interconnected, or when several companies are merged as one property. There is a good deal of discussion throughout the country to-day regarding the merits of holding company control. It is probable that holding company control will be beneficial providing it effects physical consolidation of systems, otherwise there does not seem to be much that can be said in its favor when considering the interests of the industry and consumer as paramount, particularly with respect to the larger operating companies which can afford to have on their own staffs, experts competent to solve all of their engi-

neering, construction, operating, financial, accounting, rate making, commercial and public relations problems. At the present time two holding companies function in California. As of 1925, 62 per cent. of the power of California was produced by merger companies, 25 per cent. by held companies, and 13 per cent. by municipal corporations.

FUNDAMENTALS OF POWER COST.

Going back somewhat to first principles, the cost of power delivered at load centres, whether in California or elsewhere, depends upon the capital cost of generating plants, including transmission lines and the resulting annual cost.

The capital cost depends upon many things, but as a rule is greater for hydro than for steam power.

Exclusive of taxes and management costs, the annual costs are composed of the fixed charges and running expense. Fixed charges include interest and depreciation, and running expense includes operation and maintenance, and fuel if steam plants are employed.

One of the most important fundamentals of power production cost is the relation existing between average and maximum rate at which power is necessary to answer the demand. Costs rise as this ratio or "load factor" lowers. The demand fluctuates hourly, daily, weekly and monthly, with the result that the full production capacity of a plant can be realized only over the comparatively brief period of maximum demand. Neglecting reserve capacity, which is a matter of judgment, generating facilities must be of an actual capacity to meet the maximum demand, even though this occur during but one hour of the year. Obviously then, there are many hours of the year when the investment is, in effect, but partially productive. If the demand were constant throughout the year each kilowatt of generating capacity could operate for 8760 hours and generate a like number of kilowatt-hours of energy, and the cost of each kilowatt-hour of energy would be less than that which would obtain for any other condition of operation. Thus, load factor becomes an important consideration in power economy.

The capital cost of steam plants at load centres with relatively short transmission, in general, is less than that of hydro plants with long transmission, which occasions lower fixed charges for

steam plants. Giving consideration to the cost of fuel and the relatively high running expense, however, the cost of steam-generated energy, for 100 per cent. load factor, tends to exceed the cost of hydro-generated energy. On the other hand, for low load factors, on account of the greater amount of hydro capital which is unproductively employed, and because of the small quantity of fuel required to generate the relatively small number of kilowatt-hours which obtain with low load factors, the cost of hydro-generated energy tends to exceed the cost of steam-generated energy. A low load factor and cheap fuel, then, suggest steam plant development, whereas a high load factor and expensive fuel make hydro development attractive. In either case the load factor has a determining effect on the cost of power whether it be supplied by steam plants, by hydro plants, or by a combination of both.

It follows then, with a system load factor as usual less than 100 per cent., if the load be divided into two parts, and the bottom or high load factor portion placed on hydro, and the top or low load factor portion placed on steam, that a composite economy in power production will result. This, of course, assumes that the capital cost of hydro is not so low as to preclude the use of any steam, and that fuel is not so cheap as to preclude the use of any hydro.

The capital cost per kilowatt of delivered peak of hydro plants varies greatly and will range from perhaps a minimum of $150 under very favorable conditions to $400 or more, depending largely upon storage requirements and distance of transmission.

On the other hand, the capital cost per kilowatt of delivered peak of steam plants does not vary greatly, the range being from perhaps $85 minimum to $125 maximum. In the remarks that follow a fixed capital cost of $100 is assumed as a reasonable standard steam plant cost.

The annual cost of steam plants, including the cost of "no-load" fuel, but not including the cost of energy fuel, expressed as a percentage of the capital cost thereof, is almost invariably greater than the annual cost of hydro plants similarly expressed, by approximately 50 per cent.

It is therefore possible to spend 50 per cent. more capital on a hydro plant to give the same annual cost of power from a steam plant at zero load factor.

As the load factor rises, thereby causing the annual cost of steam power to rise because fuel for energy is proportionately employed, the cost of hydro power may be kept equal to the cost of steam power at a given load factor by expending no greater amount of hydro capital per kilowatt than that which would equal the capital value of the annual cost of steam power including the cost of all fuel, capitalized at the annual percentage cost of hydro power. These general statements being true, the following principles may be established:

(1) When primary hydro power is available to suit the system load factor, and when the capital cost of a kilowatt of hydro is less than the capital value of the annual cost of a kilowatt of steam, including the no-load fuel, but not including the energy fuel, capitalized at the annual percentage cost of a kilowatt of hydro, it will be cheaper to supply all system power from hydro developments.

(2) When the cost of a kilowatt-year of hydro power is greater than that of steam power at some load factor less than the system load factor, it often will be economic in a base load primary hydro supply to furnish the total system load from both hydro and steam, using steam on the top, and the hydro on the bottom at load factors lower and higher than the given system load factor which would, respectively, obtain for the steam and the hydro.

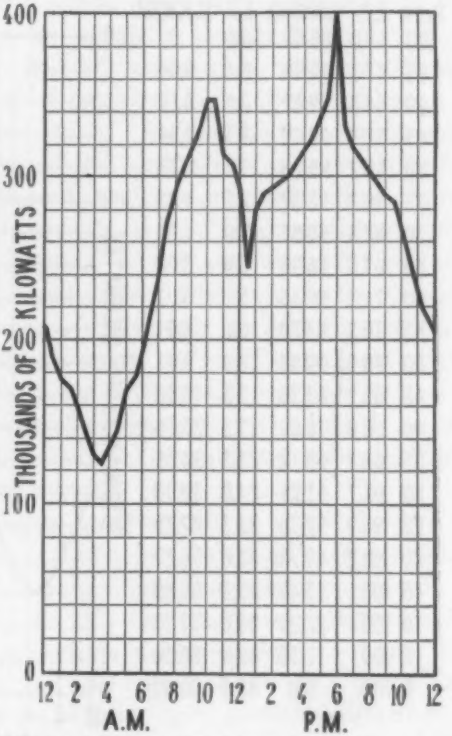
(3) When the cost of a kilowatt-year of steam power at 100 per cent. load factor is less than that of hydro power, obviously steam power will always be cheaper than hydro power at any system load factor.

(4*a*) When hydro power is available for peak from pondage but not for energy in an amount sufficient to meet the demand for energy at the given system load factor, and when this hydro peak can be furnished at a capital cost equal to or less than the capital value of the annual cost of equivalent steam power without energy fuel, capitalized at the annual percentage cost of the hydro plant, it will be economic in a base load steam supply to comb the system peak with such hydro power. The amount of this combing will be determined wholly by the ability of the hydro plant to furnish the kilowatt-hours required above the kilowatt load carried by the steam plant.

(4*b*) If at times of the year secondary energy is available from the run-of-river flow from such a hydro plant, this energy may be employed to reduce the base load energy fuel consumption, thus producing correspondingly further economy which will permit the expenditure of additional hydro capital amounting to the

DIAGRAM I.

% Peak	From Base L.F.	From Base %KWHR	From Peak L.F.	From Peak %KWHR
100	65.0	100.00	0.0	.0
95	68.5	99.95	0.7	0.05
90	72.2	99.77	1.5	0.23
85	76.2	99.39	2.7	0.61
80	80.2	98.41	5.2	1.59
75	83.8	96.47	9.2	3.53
70	86.5	92.96	15.3	7.04
65	88.3	88.20	21.9	11.80
60	90.8	83.63	26.7	16.38
55	92.8	78.37	31.3	21.63
50	94.8	72.82	35.4	27.18
45	96.8	66.93	39.1	33.07
40	98.3	60.46	42.8	39.54
35	99.5	53.52	46.6	46.48
31	100	47.61	49.4	52.39
30	100	46.07	50.2	53.93
25	100	38.40	53.5	61.60
20	100	30.71	56.4	69.29
15	100	23.04	59.0	76.96
10	100	15.36	61.2	84.64
5	100	7.68	63.2	92.32
0	100	0.0	65.0	100.00



Composite daily load curve at 65 per cent. load factor.

capital value of the fuel saved when capitalized at the annual percentage cost of the hydro plant.

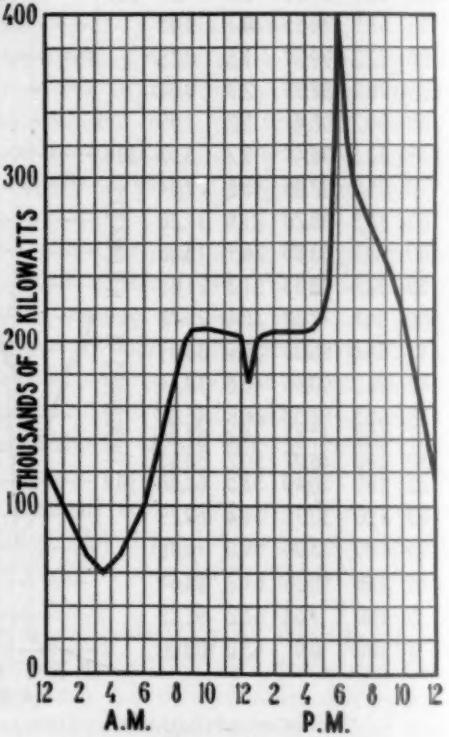
In order to give this discussion some measure of practical value, an economic study, with particular reference to the first three principles enumerated above, has been made to ascertain the cost of delivering 400,000 kw. at load centres under two load factor conditions, one for a regional supply, for which a 65 per

cent. load factor is possible, and the other for a metropolitan supply at the lower load factor of 45 per cent.

In the conduct of the studies it was necessary to ascertain the annual kilowatt-hours which lie under the annual load curve. It would be inconvenient to plot 365 daily load curves, so there

DIAGRAM II.

% Peak	From Base L.F.	From Base %KWHR	From Peak L.F.	From Peak %KWHR
100	45.0	100.00	0.0	.0
95	47.3	99.96	0.4	0.04
90	49.8	99.81	0.8	0.19
85	52.6	99.59	1.3	0.41
80	55.7	99.26	1.7	0.74
75	59.2	98.77	2.2	1.23
70	62.8	98.07	2.9	1.93
65	67.0	96.96	3.9	3.04
60	71.3	95.43	5.1	4.57
55	76.3	93.39	6.6	6.61
50	80.9	90.09	8.9	9.91
45	83.7	83.70	13.3	16.30
40	86.0	76.61	17.5	23.39
35	88.7	69.12	21.3	30.88
30	91.7	61.21	24.9	39.79
25	94.7	52.75	28.3	47.25
20	98.2	43.73	31.6	56.27
15	100.	33.41	35.2	66.59
10	100.	22.27	38.8	77.73
5	100.	11.15	42.0	88.86
0	100.	0.0	45.0	100.00



Composite daily load curve at 45 per cent. load factor.

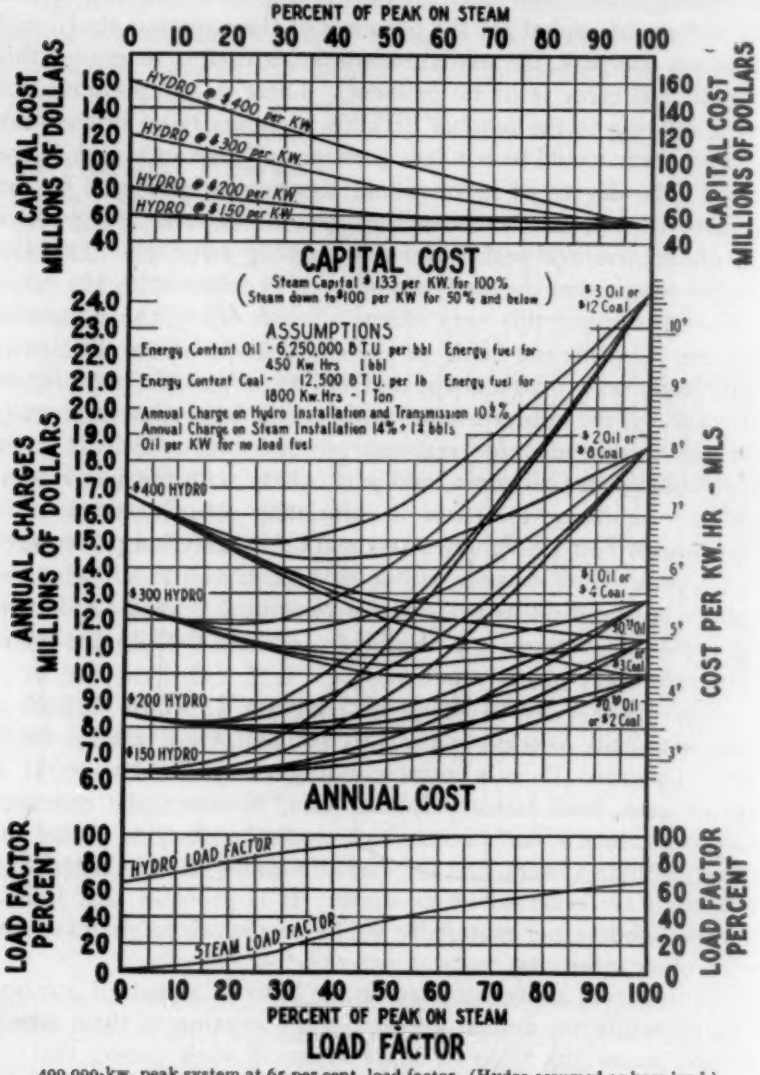
has been constructed a composite daily load curve, 365 of which would contain, with a great degree of accuracy, all of the kilowatt-hours which would be found under a yearly load curve of 365 actual days. This composite curve will also contain the peak which would have obtained on one or more occasions throughout the year in an actual load curve. The composite load curve is

therefore unavoidably inaccurate with respect to the peak because there would not be 365 peaks of the magnitude indicated in the composite curve. But since the peak is employed only to ascertain the volume of capital for the purposes of the economic study, and since but one peak, the annual maximum, is used to determine this capital, this error is of no moment. There is an appearance of error relating to the number of kilowatt-hours lying in the peak because there would be but few, perhaps only one such peak in the year. The degree of error would seem to be measured by the number of kilowatt-hours in, say, 364 such peaks lying above the annual average peak. But this seeming error does not exist for the reason that the method employed in constructing the curve took into account this very thing. So, in effect, the composite load curve, while not being of the exact shape of any particular daily load curve, is of a shape that will record, with a high degree of accuracy, the kilowatt-hours which must be produced annually for the chosen load for regional power systems such as those which obtain in California, and particularly with the larger companies. Similarly, the curve is sufficiently accurate to ascertain the kilowatt-hours which lie above and below different percentages of the peak. For instance, if one-half the kilowatt load from the bottom is placed on hydro, and the remaining one-half from the top is placed on steam, it is possible to ascertain the kilowatt-hours that each of the plants would carry.

The above remarks apply to Diagram I, which is such a composite daily load curve for a 65 per cent. load factor. Similarly, Diagram II is a corresponding composite curve for a 45 per cent. load factor. In this case, however, the endeavor was to produce a load curve which would be likely to obtain for a metropolitan system, and the load conditions of the City of San Francisco are reflected in its shape. It is probable that it is a composite curve not materially unlike one which might exist for some other large city.

Both curves are constructed on the basis of a peak of 400,000 kw., therefore the annual kilowatt-hours relating to them would be 365 times the kilowatt-hours lying in each curve, that is, 260,000 kw. for 8760 hours, or 2,277,600,000 kw.-hrs. for the 65 per cent. load factor; and 180,000 kw. for 8760 hours or 1,576,800,000 kw.-hrs. for the 45 per cent. load factor.

The curve is accurate for the two purposes for which it is used in the economic study: First, for the quantity of energy to

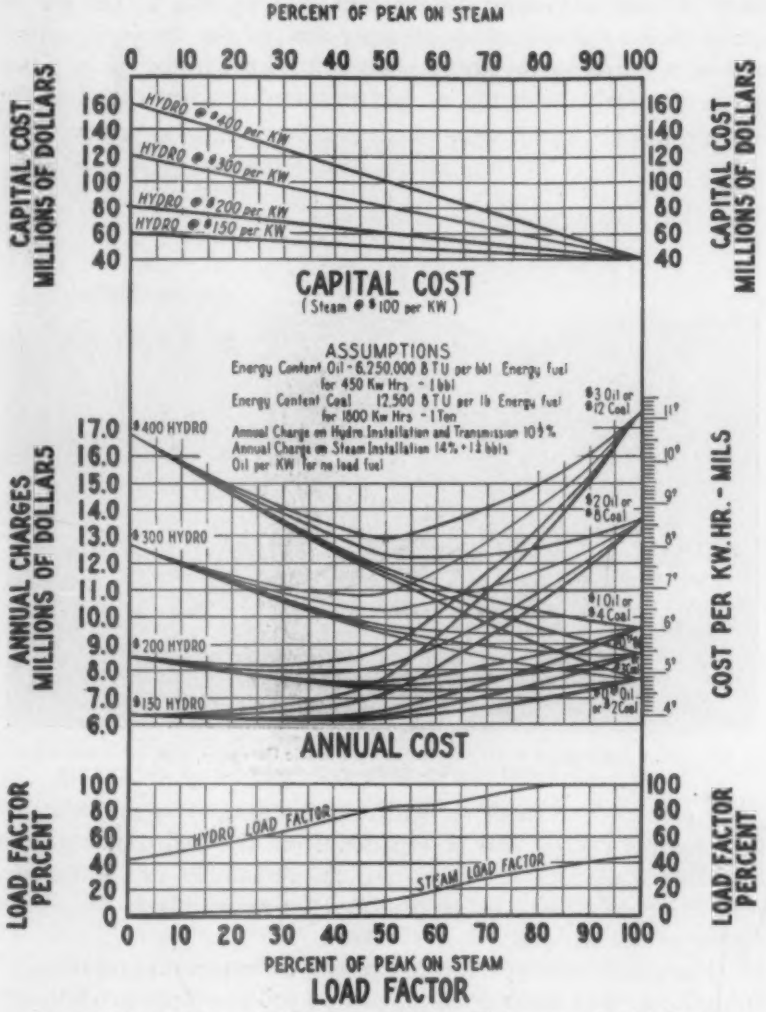

PERCENT OF PEAK ON STEAM
0 10 20 30 40 50 60 70 80 90 100
CAPITAL COST MILLIONS OF DOLLARS
160 140 120 100 80 60 40
HYDRO @ $400 per KW
HYDRO @ $300 per KW
HYDRO @ $200 per KW
HYDRO @ $150 per KW
CAPITAL COST
(Steam Capital $133 per KW. for 100% Steam down to $100 per KW for 50% and below)
ASSUMPTIONS
Energy Content Oil - 6,250,000 B.T.U. per bbl Energy fuel for 450 Kw Hrs 1 bbl
Energy Content Coal - 12,500 B.T.U. per lb Energy fuel for 1800 Kw.Hrs - 1 Ton
Annual Charge on Hydro Installation and Transmission 10½%
Annual Charge on Steam Installation 14% + 14 bbls Oil per KW for no load fuel
ANNUAL CHARGES MILLIONS OF DOLLARS
24.0 23.0 22.0 21.0 20.0 19.0 18.0 17.0 16.0 15.0 14.0 13.0 12.0 11.0 10.0 9.0 8.0 7.0 6.0
$400 HYDRO
$300 HYDRO
$200 HYDRO
$150 HYDRO
$3 Oil or $12 Coal
$2 Oil or $8 Coal
$1 Oil or $4 Coal
90¢ Oil or $3 Coal
60¢ Oil or $2 Coal
COST PER KW.HR - MILLS
10° 9° 8° 7° 6° 5° 4° 3°
ANNUAL COST
LOAD FACTOR PERCENT
100 80 60 40 20 0
HYDRO LOAD FACTOR
STEAM LOAD FACTOR
PERCENT OF PEAK ON STEAM
LOAD FACTOR

400,000-kw. peak system at 65 per cent. load factor. (Hydro assumed as base load.)

be furnished so that fuel consumption can be computed where steam power is employed; and second, for the annual peak so that

sufficient capital can be included for both hydro and steam plant capacity.

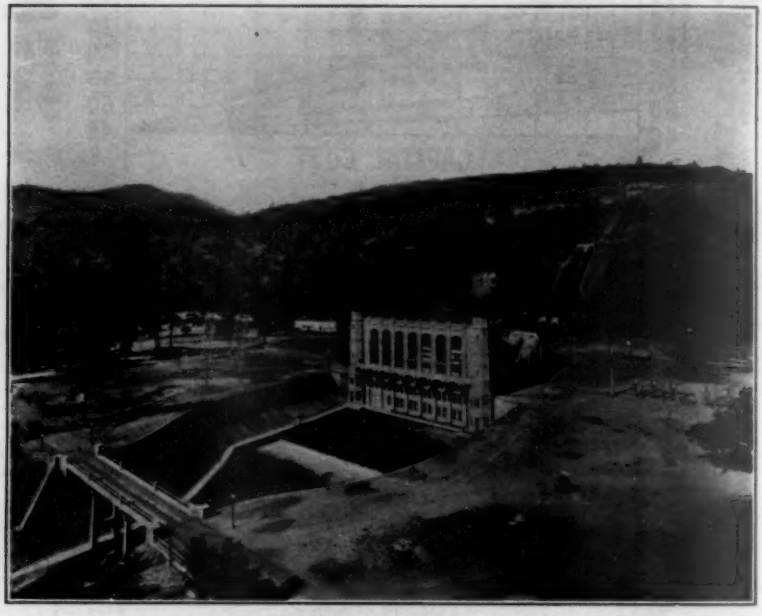
From these curves were determined the load factors and


PERCENT OF PEAK ON STEAM
CAPITAL COST MILLIONS OF DOLLARS
HYDRO @ $400 per KW
HYDRO @ $300 per KW
HYDRO @ $200 per KW
HYDRO @ $150 per KW
CAPITAL COST
(Steam @ $100 per KW)
ASSUMPTIONS
Energy Content Oil - 6,250,000 B T U per bbl Energy fuel for 450 Kw Hrs - 1 bbl
Energy Content Coal 12,500 B T U per lb Energy fuel for 1800 Kw Hrs - 1 Ton
Annual Charge on Hydro Installation and Transmission 10½%
Annual Charge on Steam Installation 14% + 12 bbls Oil per KW for no load fuel
$3 Oil or $12 Coal
$2 Oil or $8 Coal
$1 Oil or $4 Coal
$½ Oil or $2 Coal
ANNUAL CHARGES MILLIONS OF DOLLARS
$400 HYDRO
$300 HYDRO
$200 HYDRO
$150 HYDRO
COST PER KW.HR. - MILS
ANNUAL COST
LOAD FACTOR PERCENT
HYDRO LOAD FACTOR
STEAM LOAD FACTOR
PERCENT OF PEAK ON STEAM
LOAD FACTOR

400,000-kw. peak system at 45 per cent. load factor. (Hydro assumed as base load.)

kilowatt-hours from base and peak for various percentages of the system peak from 100 per cent. to zero. This information is given on the curves. The 65 per cent. curve indicates, for

instance, that if 60 per cent. of the annual peak from base is carried on hydro plants, 83.6 per cent. of the required energy will be supplied from such hydro plants on a load factor of 90.8 per cent. If the remaining 40 per cent. of the peak is carried on steam plants, the balance, or 16.4 per cent. of the required energy, will be supplied by the steam plants on a load factor of 26.7 per cent. Similarly, with the 45 per cent. curve, when carrying the



Pit River No. 1 power house of the Pacific Gas and Electric Company, a typical low head plant. Head, 454 feet. Capacity, 70,000 kv.-a.

same per cent. of peak on hydro and steam, 95.4 per cent. of the required energy will be supplied from the hydro plants on a load factor of 71.3 per cent., and the remainder of the energy, or 4.6 per cent., will be supplied by the steam plants on a load factor of 5.1 per cent.

The capital cost of hydro, including transmission and terminal substations, was taken as $150, $200, $300 and $400 per kilowatt of delivered peak, which units are intended to include the cost of storage works to effect stream regulation and to carry over a water supply for dry years, but not spare capacity. The capital cost of steam plants was taken at $100 per kilowatt plus a capital

cost for back transmission for the 65 per cent. load factor load, varying from $33 per kilowatt for 100 per cent. steam supply to nothing for a 50 per cent. steam supply; and $100 per kilowatt for the 45 per cent. load factor load, no back transmission being considered in this case because it is assumed that a load of 45 per cent. load factor, in the nature of things, would be metropolitan.

Annual costs, on the basis of low-maintenance hydro construction and modern steam plants, were derived by applying to the capital the following annual percentages:

	Hydro Delivered. %	Steam. %	Steam Power Transmission. %
Return	8	8	8
Depreciation	1	3	1
Operation and maintenance	1.5	3	1.5
Total per cent. of capital	10.5	14	10.5

In addition to the above the cost of 1¼ barrels of oil per kilowatt per year was included with annual steam plant cost.

No losses were assumed in computing the volume of hydro capital for the reason that the unit capital costs are considered as those which would effect the delivery of 1 kw. at the load centre. With the steam in the 65 per cent. load factor case, however, the capital and fuel consumption were computed on the basis of a 3 per cent. loss for steam loads of 100 per cent., to zero loss for steam loads of 50 per cent.

A barrel of oil was considered to contain approximately 6,250,000 heat units and the energy yield was taken at 450 kw.-hrs. when burned in modern high-pressure, high-temperature steam-generating plants, after the "no-load" fuel has been supplied. In the economic study, oil prices were chosen at $.50, $.75, $1, $2 and $3 per barrel, which is equivalent to $2, $3, $4, $8 and $12 coal at 12,500 B.t.u. per pound. Under the assumption of 450 kw.-hrs. per barrel, plus the "no-load" fuel of 1¼ barrels per kilowatt-year, the steam plant economy would be 14,770 B.t.u. per kilowatt-hour, which is customary steam plant performance in efficient modern plants. The annual cost of serving the total load was determined for different divisions of the peak on steam and hydro, commencing with zero per cent. on steam and advancing in 10 per cent. units to 100 per cent. The total cost of

fuel was ascertained from the information given on the load curve. The whole was plotted as a family of curves for the different capital costs of hydro and the different fuel prices for the

Lake Britton reservoir of the Pacific Gas and Electric Company with the Pit River No. 3 diversion dam at the lower end.

two load factor conditions. These curves are offered as simple methodic expressions of the cost of carrying the load in various ways. They indicate the total capital requirement, the annual cost, the kilowatt-hour cost, and the load factor for the steam and

hydro for all of the chosen unit capital costs and fuel prices, for different percentages of the peak which are carried by steam from zero per cent. to 100 per cent. For instance, when the capital cost of hydro is $300 per kilowatt, and the cost of oil is $2 per barrel, the cheapest way to serve the 65 per cent. load factor load is to deliver 300,000 kw. of hydro and 100,000 kw. of steam power. To do this $100,000,000 of capital will be required. The annual

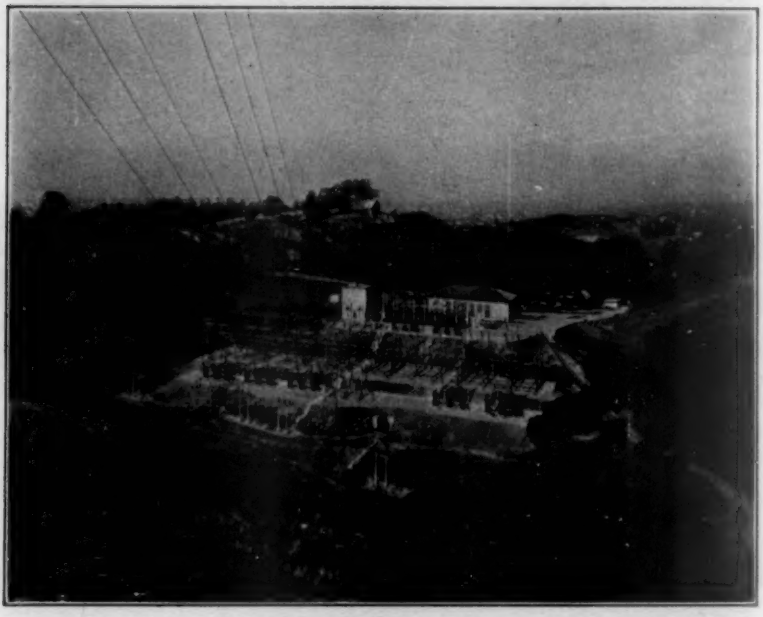
Drum power house of the Pacific Gas and Electric Company, a typical high head plant Capacity, 37,500 kv.-a., now being increased to 50,000 kv.-a. Head, 1357 feet.

cost will be $11,500,000, and the cost per kilowatt-hour slightly above 5.2 mils. The hydro will operate at 84 per cent. load factor, and the steam at 9 per cent.

Turning to the other set of curves, we observe that, under the same conditions of hydro capital cost and fuel prices, the cheapest way to serve the 45 per cent. load factor load is to deliver 200,000 kw. of hydro power and 200,000 kw. of steam power. In this case the required capital is $80,000,000, the annual cost $10,300,000, and the cost per kilowatt-hour 6.5 mils. The hydro will operate at 81 per cent. load factor, and the steam 10 per cent. load factor.

The curves are illuminating in many other respects, among which the following are cited:

With hydro capital at $150 and oil at $1 per barrel, and under, the load may be carried exclusively on hydro, or up to 20 per cent. on steam for 65 per cent. load factor, and up to perhaps 40 per cent. for the 45 per cent. load factor, without material change in the economy. On the other hand, with hydro at $400



Claremont Substation of the Pacific Gas and Electric Company, a 110-kv. terminal station on the outskirts of Oakland, Calif.

per kilowatt, and oil at $1 per barrel, the 65 per cent. load factor load may be carried 100 per cent. down to 60 per cent. on steam at practically the same economy. For oil prices less than $1 it would be economic to carry all of the load on steam; with $2 oil, from 30 to 50 per cent.; and with $3 oil, 25 per cent. In the 45 per cent. load factor case, however, it would be economic to carry all of the load on steam with oil at $1 per barrel and under; from 50 to 70 per cent. with $2 oil; and 50 per cent. with $3 oil.

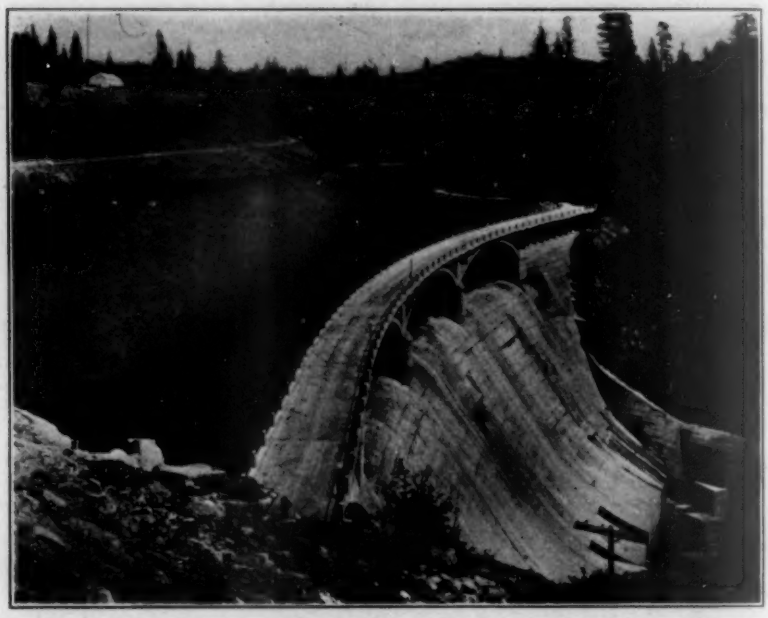
The foregoing study is a practical application of economic principles in an attempt to ascertain the least annual cost of fur-

nishing a power supply taking into account the cost and price variables that exist with water power and fuel. Under some conditions, in the adjustment of higher fixed charges and low operating coats of hydro with lower fixed charges and higher operating costs of steam plants to keep the total annual cost a minimum, the study recalls Kelvin's Law. Obviously it is theoretical in that it does not, nor can any general study, take account

Vaca-Dixon Substation of the Pacific Gas and Electric Company, a 220-kv. terminal station housing synchronous condensers, from which power from the Pit River is distributed to the San Francisco Bay region.

of the special factors which might obtain in an individual situation of a power company contemplating an extension of facilities. It presupposes economic, social and political freedom in the choice of natural resources when more than one is available for use in power generation. Obviously the power supply for any region will be produced from the resources present, and its cost to a large extent will be that which results from the working of economic processes. It may be, with both fuel and water power at hand, that the cost of a unit of power from each, although reasonable as reasonable cost may be measured by the value of the service rendered, will be higher than that which would result from

the use of both resources. The curves indicate that with certain relations between the capital cost of water power and fuel prices, although the cost of all hydro is less than the cost of all steam, the cost of combined hydro and steam is even less than the cost of all hydro, and conversely, with certain other relations, that although the cost of all steam is less than the cost of all hydro, the cost of combined steam and hydro is even less than the cost



Diversion dam of Pit River No. 3 development of the Pacific Gas and Electric Company forming Lake Britton. Constructed in 1925. Height of dam, 112 feet, foundation to Spill Crest. Capacity of reservoir, 32,500 acre-feet. Effective for regulation, 8000 acre-feet.

of all steam. Under such conditions both power supplies must be used if a maximum of economy is to be enjoyed.

The study, while perhaps not being quantitatively perfect, develops the economic certainty that at times the cost of power will be a minimum from joint steam and hydro operation and is fairly accurate in indicating the direction in which economy lies; or, said in another way, it points out the path to maximum economy in power generation when both fuel and water power are available, although either one may be exclusively employed at a reasonable cost, or at a cost less than that obtaining with the other.

CONCLUSION.

In concluding this narrative of the power situation in California, one may venture the opinion that the western utilities are ever alert in looking to the extension and cost of their service. Their success is born of the success of general industry, and this will be maintained only by having power available everywhere at a cost to the ultimate consumer which is in agreement with the most economic production feasible with the resources at hand, and at the present time in conformity with the lowering trend in commodity and money prices, although at no time, even during the war and post-war period, has electric service had an advance in price at all in proportion with the rise in the commodity market. The engineer in the central station industry, to be effective in his contribution to the organized effort and team work which are essential to the progressive advance of the industry, must be gregarious and view his problem broadly, taking account not only of its technical features, but also of the economic and social elements which enter. Then will his engineering be sound in the broadest sense.

Among other things, he must look to the changed situation of higher hydro and lower steam costs, to the end that there will be a continuance of an ample supply of low-priced power. The spectacular hydro development with its high voltage transmission is justified if it produces power at a cost which is no greater than that which would obtain with steam power, or with coördinated water and steam power. The economy and utility of combined hydro and steam generation are being studied, for it is dependable central station power at the lowest present and future cost, as far as this latter can be determined, that is the prime objective of the engineer.

The situation in other Pacific Coast states is not materially unlike that of California, both historically and in trend. The problem is essentially an economic one in which the hydraulically powered West is holding its cost within range by the introduction of a suitable measure of steam power, even as will the steam-powered East by the use of a justifiable proportion of hydro power.

The economic study which is submitted with this account is purposely sheared of all the intricacies of technical detail, so that the economic principles of combined hydro and steam power may

be generalized and readily perceived. The study may be employed to explore for the border-line cases upon which detailed attention needs be focussed. It will be seen from the two groups of curves that the cost range of minimum cost with a high load factor is considerably less than that with a low load factor. Furthermore, the curves show that while there is often less capital required to carry a low than a high load factor load, the kilowatt-hour cost of the former is considerably higher.

While the curves are eloquent in emphasizing the axiom of high load factor, what is of equal or perhaps greater importance is high capacity factor. This is gained by having the system installed generating capacity as little in excess of the system peak demand as service and other conditions will allow, and by having such capacity operate on the highest possible load factor. Obviously a high capacity factor has the tendency to effect a downward trend in both capital requirements and consumers' rates.

The trend of the future in the central station industry throughout the country will be the continued consolidation of diverse local markets and the further interconnection of regional power-producing facilities, in the interests of high load factor and low-priced power. With pardonable pride it might be emphasized that the great expansion in the electrical industry on the Pacific Coast, the economies which it has realized, and the other advantages which have accrued, are the result of regional coördination of power demand and power supply under private initiative. It is to be hoped that the public will not lose itself in the mystic maze of public ownership, with its consequent checking of industrial progress. The electrical industry is still young, is highly technical, and is an agency of production. Franklin said, "God gives all things to industry," but no industry, and particularly the electrical industry, can advance and flourish under the weight of government ownership and operation; public opinion should be such as will not only permit but encourage it to continue that unhampered course in the future which it has pursued in the past with such gratifying results in the public and its own interest alike.

MATHEMATICAL METHODS OF FREQUENCY ANALYSIS OF SIZE OF PARTICLES.*

PART I. METHODS.

BY

R. P. LOVELAND and A. P. H. TRIVELLI.

A SIZE-FREQUENCY analysis of a system of dispersed particles is usually made either to give explicit expression to the comparison of the materials or to discover fundamental relationships connected with the distribution. To do this we must eliminate the haphazard fluctuations, which may be done by fitting to it some formula that some reasonable criterion indicates is representative. We then also have an expression whose parameters are of much utility as explicit constants for comparison and study. The relevancy of size-frequency to photographic research has been previously discussed.[4] It is now a well-recognized fact that the size distribution of the grains is one, although *only* one, of the determining factors controlling the speed, contrast, latitude, resolving power, etc., of the photographic plate. The method of analysis must be a single consistent system to serve the purpose. Although the literature dealing with size-frequency curves is voluminous in other fields, especially biometrics, it is scant in the application to particle-size distribution which possesses some peculiar problems. A survey discovered no single frequency system that was generally applicable, so an investigation was made to form such a one. It is felt that the methods adopted should be useful for the size-frequency analysis of other plant and laboratory products.

SIZE-FREQUENCY, NATURE AND DETERMINATION.

Our size-frequency observations consist of a series of discrete approximations, y_1, y_2, y_3, etc., where the latter may be considered as rectangular areas of a histogram. These *a posteriori* probabilities of occurrence of grain sizes $Nf(x_i)\,\Delta x$ (where N is the total number observed, Δx the class width chosen, and $f(x_i)$ an observed fraction) are prevented from closely approaching the

* Communication No. 300 from the Research Laboratory of the Eastman Kodak Company.

a priori probability law, $\Omega(x)dx$, solely by the limits of experiment.

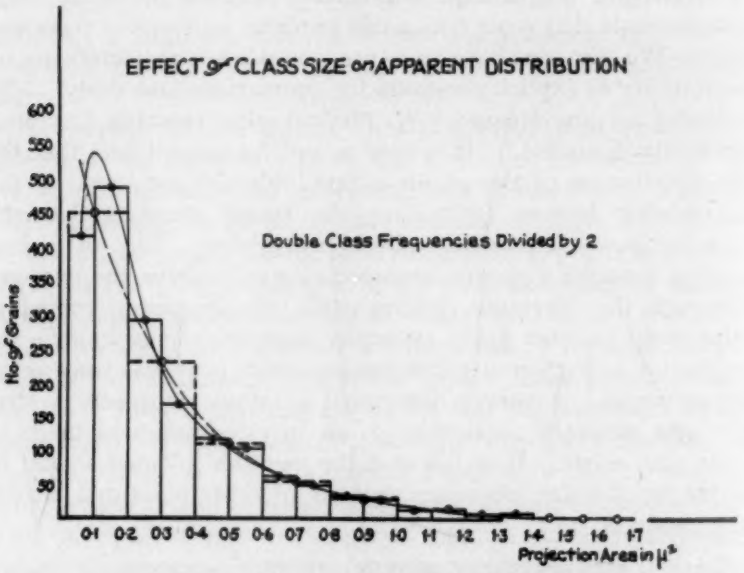
The details of the experimental method used in this laboratory have been given in a previous paper.[6] Essentially, it is the direct optical one of measuring the photographed images of the grains magnified 10,000 X.

LIMITATIONS IMPOSED BY EXPERIMENT.

The experimental errors of this method were discussed in the same paper.

The width of the classes necessitated by the practical requirements of measurement is often a serious limit to the correctness of

FIG. 1.



the representation. This may be illustrated by doubling the class width of a measured distribution. The new mid-class points are a linear average of each previous pair and may deviate considerably from the curve previously obtained. In fact, with skew distributions the class width can always be widened to such an extent as to hide the maximum and convert the data apparently to that of the modeless *L* type (Fig. 1). For accurate representation of skew dispersions the class width must be chosen so

as not to seriously distort the position and character of the mode. This may necessitate large samples for measurement. This increased labor of greater measurements or the alternative of a marked increase of the haphazard fluctuations fixes the lower limit to the restriction of class width.

Often the physical limitations of the method of measurement curtail one end of the distribution and produce irregular class widths, notably when the lower limit is greater than zero, as in the microscopical method which is limited by optical resolution. These variations in class width must be taken into account in plotting the frequency curve. $f_c = f_{obs.} \frac{\overline{\Delta x}}{\Delta x}$, where f_c is the frequency corrected from $f_{obs.}$, the observed value of the variant class $\overline{\Delta x}$, to that of the standard class width Δx. Whenever possible the class widths should be constant and the complete distribution recorded, for only then can the integral of the distribution be obtained, which is absolutely essential for some of the most common methods of treating data.

This optical truncation of the lower extremity of the observed distribution, with microscopic measurement, acts like a curtain hiding the nature of the curve below that point. The lower limit of the observed portion of the first class is very indefinite. The images of grains that are at the lower limit of resolution do not suddenly disappear from the photomicrographs (or even decrease in size for the smaller grains) but merely become lighter and lighter gray patches, so that many grains are included in the classification that are below the limit of actual resolution. The theoretical limit of resolution with the Wratten C filter used is about 0.15 micron or 0.018 square micron.

The actual *resolving* power is never as great. In spite of the phenomenon mentioned, it is obvious that particles cannot be counted down to zero size. The lower limit finally adopted is 0.02 μ^2, or practically theoretical resolving power which made the data considerably more consistent. Certainly this lower limit lies within the extremes of 0.01 μ^2 – 0.03 μ^2, which fixes the upper limits of error of the first class not to exceed 15 per cent. for this cause alone. Fixing an arbitrary lower limit to the first class width would entail more labor than it is worth since at present the obviously smallest grains need no measurement.

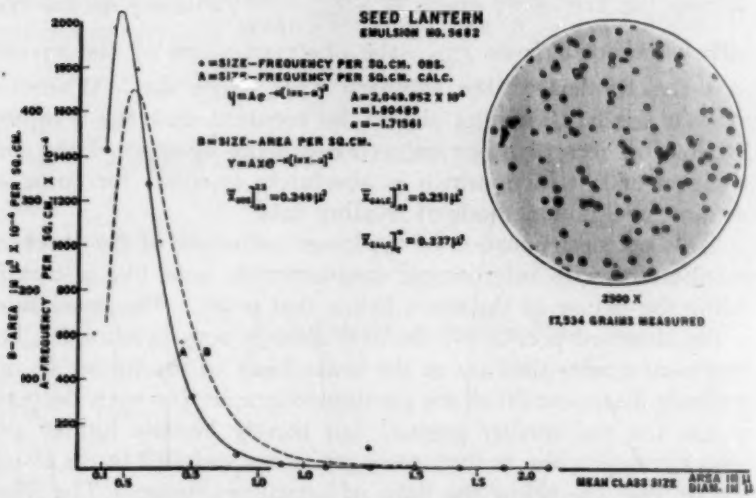
The truncation of the frequency curves, preventing their total

integration, presented a problem which was finally solved by graphical methods, developed while attempting to obtain a formula to fit some observed data.

APPLICATION OF DOUBLE LOG PAPER TO SIZE-FREQUENCY.

The method of attempting to fit all frequency curves with either the L-shaped inverse exponential or the symmetrical "normal" Gaussian formulas is an utter failure as was well demonstrated by a certain photographic emulsion.[7] Since plotting its size-frequency on double log paper caused the distribution to

FIG. 2.



resemble an inverted parabola, the corresponding formula was applied (Fig. 2). The fit was splendid, but one disconcerting fact was noted. The calculated curve showed a maximum very much higher than seemed justified by the observed data. A quick measurement was made with half of the previous class width, *i.e.*, with $\Delta x = 0.1\ \mu^2$. The results lay remarkably close to the curve calculated from the data of the coarser grain size as the small crosses in Fig. 2 show. Another determination of a different emulsion number gave an equally excellent fit with very similar parameters.

Double log paper has proven to be an extremely useful instrument in size-frequency analysis. Some of its advantages might be enumerated.

(*a*) It removes the inflection point. This usually simplifies the curve to the eye and makes it easier to draw in a likely curve. It may transform it into a recognizable type.

(*b*) The locus of the maximum point (Pearson's mode) is unchanged and becomes much easier to determine graphically.

(*c*) Plotting back values read from the double log curve onto ordinary graph paper often seems to give the most representative curve obtainable otherwise than by calculated graduation.

(*d*) It becomes possible and reasonable to make the short extrapolation desired in order to obtain the frequency of the first class with zero as a lower limit, by reading off the value of the frequency at $x = 0.05$. This makes available as a rather close approximation, the total integral and all of the functions connected with it. The error must be considerably less than by neglecting this extrapolation and it is probably rather small, granted the assumption of continuity of the curve within the first class.

(*e*) It is quick and simple. No calculations are required.

(*f*) Simple relationships may become apparent that were not otherwise obvious. This will be illustrated in the second paper.

METHODS OF FITTING FORMULAS.

The two principal general methods of fitting data with formulas are:

(*A*) Gauss' method of least squares.[33, 38]

(*B*) Pearson's method of moments.[13]

Both are too well known to need description.

(*A*) The first fits the curve by the most accepted criterion—that the sums of the squares of the deviations should be a minimum. Sometimes it is the only method that can be used. On the other hand, except for linear equations, it is only an approximation method. For non-linear equations its facility depends upon the rapidity of convergence of Taylor's series. In some instances, therefore, the method cannot be used at all.

(*B*) Pearson's method of moments, which was invented in order to apply his frequency system, fits the curves by equating as many moments of the observed data to those of the computed curve as there are constants to be determined, where the nth observed moment is $\Sigma y x^n$ and the theoretical moment is the

integral, $\int yx^n dx$. Finite class width should be taken into account since the method involves integration. In fact it fails entirely if:

(*a*) The complete distribution is not known.

(*b*) The class sizes are irregular.

(*c*) The frequency distributions are extremely skew.

Since the errors of the observed moments increase rapidly with their degree, there is a natural limit to the number of parameters that can be determined.

Since this method is, in general, more rapid than that of least squares it was hoped that it could be used to sufficient accuracy by completing the first class-frequency through extrapolation on the double log graph.

SELECTION OF MATHEMATICAL EXPRESSION FOR FREQUENCY DISTRIBUTION.

Most of the formulas proposed for application to frequency distributions may be grouped into three general schools.

(I) Systems developed from the geometrical definition of an equation.

(*a*) Method of Krichewsky.

(*b*) Pearson's system of enclosed formulas.

(II) Formulas or systems developed by analysis of the relationship of the empirical distribution to that of the "normal" LaPlace-Gaussian law. Methods of translation.

(III) Generalization through series development. Generating function adjusted by its derivatives.

Other miscellaneous and purely empirical mathematical formulas can and have been applied.[1]

All of these methods were examined using, as a common basis of test, the size-frequency data of a precipitation of silver bromide, a5c prepared by Doctor Lambert of these laboratories. It was chosen because of its marked skewness yet definite inflection point.

I*a*. *Method of Krichewsky.*—The method of Krichewsky [8] is one involving graphical integration and does not generally lead to mathematical formulation. It failed in this case and will not be discussed.

I*b*. *Pearsonian System.*—Considerable attention was paid to the Pearsonian system because of its wide use and its vigorous and prolific originator's claims [11] for its universality and theoreti-

cal validity. It consists of a system of about a dozen closed frequency formulas with a criterion, based on the moments of the observed data for indicating which should be used. In fact, the method of moments is so intricately interlinked with this system that both are affected by the same deficiencies.

This criterion not only indicated that the size-frequency of $\alpha 5C$ should be fitted by his type I, but for all such precipitations and probably all emulsions.

$$y = y_0 \left(1 + \frac{x}{a_1}\right)^{-m_1} \left(1 - \frac{x}{a_2}\right)^{m_2}. \qquad \text{Form 1.}$$

Such a result was necessary for our purposes. Type I is the most general of his formulas and seemed quite suitable. Application by the method of moments failed utterly, however, due to the skewness of the distribution.

An attempt was made to apply it by the method of least squares using the transformation

$$y = A\,(x - a_1)^{m_1}\,(a_2 - x)^{m_2}$$

where

$$A = y_0 a_1^{-m_1} a_2^{-m_2} \qquad \text{Form 2}a.$$

and also the form

$$y = Ax^{m_1}\,(a_2 - x)^{m_2}. \qquad \text{Form 2}b.$$

This also failed due to the slowness of the convergence of the a_2 terms in Taylor's series,

$$\frac{\partial r_y}{\partial a_2} = \frac{n\,(n-1)\cdots(n-r+1)}{(a_2 - x)} \cdot y\,.$$

Therefore this popular method was also abandoned.

II. *Methods of Translation from the Normal Law.*—In 1879 McAlister assumed that[17] the geometric mean instead of the arithmetic mean might be the most representative value of a series and obtained the logarithmically transformed Gaussian formula we used to fit Seed Lantern data. (Fig. 2.) Since then many others[18, 20, 21] have suggested empirically generalizing the normal law by some specified transformation of variable, including Rietz's suggestion[27] to substitute kx^n for x to fit certain skew curves.

Kapteyn and Van Uven, of Groningen, finally developed[19, 24, 25, 26] such a system from theoretical considerations.

The second and most valuable paper of this series seems to be generally unknown.

They assert that only those attributes directly affected by the natural causes of growth or production will be distributed according to the Gaussian Normal Law. All others will be functions of these and unless the function is linear the distribution will be skew. Conversely, after extended theoretical consideration of the conditions of growth, they claim that if x is an observed attribute, asymmetrically distributed, there will be some other quantity, z, that is normally spread as

$$\frac{1}{\sqrt{\pi}} e^{-z^2} dz \tag{1}$$

and its relation to x, $z = f(x)$ will describe the growth of x. Then also, they show the relation $\psi(x) = \frac{1}{f'(x)}$ would express the effect of the causes of growth on the attribute measured. The correspondence (x, z) may be expressed by the relation

$$\int_{x_1}^{x} y dx = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{z} e^{-z^2} dz = \theta(z). \tag{2}$$

Van Uven proved that, while the probable error, ρ, of z is $\rho_z = \frac{0.6}{\sqrt{N}}$ at $z = 0$, ρ increases with increase of z so that "at the limits the value of z is absolutely uncertain, so that it is impossible to decide which value of x_0 or x_n (the lower or upper limit of x) must be made to correspond with $z = -\infty$ or $z = \infty$."

For the practical determination of the normal function $z = f(x)$ we must use the discrete values $N \int_{x_1}^{x_2} \Omega(x) dx = y_{x_1}^{x_2}$, assuming the *a posteriori* probability, p, to be equal to the *a priori* probability $\theta(z)$ (inverse Bernoulli theorem) which by (2) gives:

$$p = \frac{1}{N} (y_1 + y_2 + y_3 + + + y_i) = \theta(z), \tag{3}$$

where y_1, y_2, y_i . , etc., and the observed frequencies of x within the class range $x_{i+\zeta} - x_{i+\zeta}$. Having formed the fractions p_1, p_2, $-p_i$, the value of the corresponding z_i can then be read from an ordinary table of the probability function $\theta(z)$ and plotted against x.

Whereas Kapteyn and Van Uven assume the probability integral to be written as $\theta(z) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{z} e^{-z^2} dz$ in their tables, the usual tables give the values

$$\theta(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{t} e^{-\frac{t^2}{2}} dt, \tag{4}$$

or

$$\phi(t) = \frac{1}{\sqrt{2\pi}} \int_{0}^{t} e^{-\frac{t^2}{2}} dt, \tag{5}$$

where $t = \sqrt{2z}$.

It is most often convenient to give the normal function as the relation of the observed distribution to $\theta(t)$. obtaining a function proportional to the one originally proposed. Jørgensen gives very complete tables of form (4) for positive values of t. If $t<0$, that is, if $p<0.05$, the values of t can be read except for sign by using values of $1-p$. When the tables are given in form (5) as by Glover,[21] the values of t will correspond to $p-0.05$ and have the same sign.

The normal function may be obtained graphically by plotting the summated frequencies on probability paper. Unfortunately, the only brand of probability paper known to the writers, that sold by the Codex Book Company, has the independent variable x plotted as ordinates. It is, therefore, necessary to look through the paper to perceive the ordinary mathematical relationship.

The equation of the corresponding frequency curve is

$$y = \frac{f'(x)}{\sqrt{\pi}} e^{-[f(x)]^2}, \tag{6a}$$

or

$$y = \frac{\vartheta'(x)}{\sqrt{2\pi}} e^{-\left[\frac{\vartheta(x)}{2}\right]^2} \text{ for } t = \vartheta(x) = \sqrt{2f(x)}. \tag{6b}$$

It may be very difficult or impossible to deduce the equation of the normal function from its graph.

Pearson severely criticized this method "since the new unknown, z, leaves one no further advanced than the original frequency generality $y=\Omega(x)$." [23] He further criticized [22] the creation of the "shadow variable" which "is introduced solely to reduce the frequency by hook or crook to that fetish distribution, the Gaussian curve." Moreover, from Kapteyn's fundamental assumption Pearson derived the general differential equation

$$\frac{1}{y}\frac{dy}{dx} = \frac{-x}{\sigma^2 F(x)} \tag{7}$$

with $F(x)$ rather than Kapteyn's $f(x)$ to be determined. Pearson assumed $F(x)=c_0+c_1x+c_2x^2$ in order to derive his own system.

The explicit relationship between the two systems can easily be found by differentiating (6a) and equating it to (7),

$$F(x) = -\frac{\sigma^2 x f'(x)}{f''(x) - 2f(x)[f'(x)]^2}. \tag{8}$$

Now there is no method by which $F(x)$ may be determined. Simple forms of $f(x)$ may and do occur which when transformed to $F(x)$ cannot be adequately expressed as a quadratic and hence by the Pearsonian system. Such a form is $f(x) = \lambda \log x$. We feel that the normal function of Kapteyn furnishes an exceedingly valuable and practical analytical tool entirely aside from the theory used in its development.

APPLICATION OF NORMAL FUNCTION SYSTEM.

Since this is an integration method, extrapolation on the double log graph is again necessary. Test showed that there is little danger of this affecting the type of the normal function.

TABLE I.

Seed Lantern, No. 6102. Normal Function.

Class.	Upper Class Limit in μ^2.	y = No. of Grains $\times 10^{-4}$ in Class.	Σy_i	$p = \frac{\Sigma y_i}{N}$	z_i
1	0.2	995.5	995.5	0.3531	− 0.267
2	0.4	1143.3	2138.8	0.7586	+ 0.496
3	0.6	438.2	2577.0	0.9140	0.966
4	0.8	138.4	2715.4	0.9631	1.264
5	1.0	60.4	2775.8	0.9845	1.525
6	1.2	22.3	2798.1	0.9924	1.717
7	1.4	9.8	2807.9	0.9959	1.87
8	1.6	4.8	2812.7	0.9976	1.99
9	1.8	3.35	2816.05	0.9988	2.15
10	2.0	2.00	2818.05	0.9995	2.33
11	2.2	0.74	2818.79	0.9998	2.5
12–16	2.4–3.2	0.63	2819.42	1.0000	

The normal functions of the Seed Lantern emulsions were first obtained. When plotted on semi-log paper the nature of the distribution is unmistakable. Using Van Uven's notation,[24] the normal function is

$$z = \lambda (lnx - lnx_m) \qquad (9)$$

where $lnx = \log_e x$ and x_m is the median of the distribution where $z = 0$. The proportionate reaction function is $\eta = x/\lambda$. Thus simply in this case, the properties of the distribution become completely known.. The equation for the frequency curve is

$$y = N \frac{\lambda}{\sqrt{\pi x}} e^{-\lambda^2 (lnx - lnx_m)^2} \Delta x, \qquad \text{Form 3.}$$

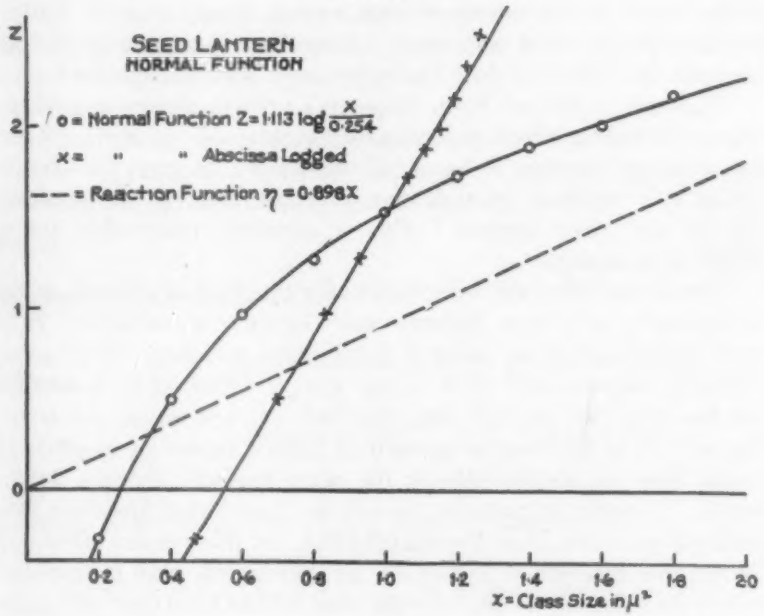
where Δx is the class width usually taken as unity,

Let

$$A_1 = N\frac{\lambda}{\sqrt{\pi}} \qquad \alpha_1 = lnx_m \qquad \kappa = \lambda^2$$

$$y = \frac{A_1}{x}\, e^{-\kappa (lnx - \alpha_1)^2} \qquad \text{Form } 3a.$$

FIG. 3.



Then

$$lny = lnA_1 - lnx - \kappa\,(lnx - \alpha_1)^2$$

$$= lnA_1 - \kappa\,(lnx)^2 - (2\kappa\alpha_1 - 1)\,lnx - \kappa\alpha_1^2$$

$$= lnA_1 - \kappa\left[(lnx)^2 - 2\left(\alpha_1 - \frac{1}{2\kappa}\right)lnx + \left(\alpha_1 - \frac{1}{2\kappa}\right)^2\right]$$

$$+ \kappa\alpha_1^2 - \alpha_1 + \frac{1}{4}\kappa - \kappa\alpha_1^2$$

Let $\quad lnA = lnA_1 - \alpha_1 + \frac{1}{4}\kappa$ and $\alpha' = \alpha_1 - \frac{1}{2}\kappa$

$$y = Ae^{-\kappa (lnx - \alpha_1)^2} \qquad \text{Form } 4a.$$

It is the peculiar property of this function that by similar development one may go from formula 4 to

$$y = A_{11}x^n e^{-\alpha(\ln x - \alpha_{11})^2}, \qquad (10)$$

where n is any positive *or* negative integer, by a mere change of the constants A and α. This unique property of this formula means that whether we measured diameters, areas, volumes or weights, we would obtain the same distribution law, differing only in the values of the constants since we can simply make a change of variable $t = cx^n$ in each case. Since this is true only of this formula this observed fact had previously been unexplained.

Formula 3 derived from Kapteyn's growth theory is equivalent to formula 4 which was empirically obtained (p. 200). Since the reaction function is linear, if the theory be true, the distribution is a result of growth that is proportional to the attained size at any given instant. This is certainly reasonable for a chemical precipitate.

The error due to the substitution for integration of summation of ordinates at a finite distance apart is quite appreciable. It is most easily studied by using a distribution calculated from some assumed formula such as $z = \lambda \log x/x_m$. Division by a smaller number than the correct total number, N, will cause the semi-logged line to be concave upward at higher values of x while if larger than the correct number the curve becomes concave downwards. Usually in practice, it will be found that the first few points slope down from the straight line, for this reason. Division by the true integral N, if the p's are summated, may give worse values than does $N_0 = \Sigma y$. Using class widths of $0.01\mu^2$ the parameters of the frequency formula 3 have been determined from the normal function to one in the third place.

$$\lambda = \frac{z_2 - z_1}{\ln x_2 - \ln x_1}$$

$$x_m = \text{abscissa of intercept at } z = 0. \qquad (11)$$

Statistical fluctuations tend to be smoothed out in the process of summation. In fact, after a deviation of 100 per cent. from the correct frequency of the first class the curve comes back very close to the original within a few succeeding classes. As the errors of this class certainly do not reach this magnitude, they should not obscure the nature of the function.

When the normal function of an experimental emulsion, $W12C_2$, was plotted on semi-log paper, it seemed to curve up from the straight line. A constant x_0, defining the lower limit of size, was introduced, as suggested by Van Uven,[24] giving the normal function

$$z = \lambda ln \frac{x - x_0}{x_m - x_0} . \qquad (9a)$$

Its corresponding generalized frequency curve gave a good fit, when applied to the frequency data by least squares, with a value of $x_0 = 0.117$.† A negative value of x_0 can have no physical meaning.

Since this type of formula seems to be particularly valuable for the expression of the size-frequency of precipitates, its forms will be collected and the conversion factors given.‡

Form. Form.

$$3°. \quad y = N \cdot \frac{\lambda}{\sqrt{\pi}(x - x_0)} e^{-\lambda^2 [ln (x - x_0) - ln (x_m - x_0)]^2}$$

$$3a°. \quad y = \frac{A_1}{x - x_0} e^{-\kappa [ln (x - x_0) - \alpha_1]^2} \qquad y = Ae^{-\kappa [ln (x - x_0) - \alpha_3]^2} \quad 4a°.$$

$$3b°. \quad y = \frac{A_1}{x - x_0} e^{-k [\log (x - x_0) - \alpha_2]^2} \qquad y = Ae^{-k [\log (x - x_0) - \alpha]^2} \quad 4b°.$$

$$\log A_1 = \log A + \alpha + \frac{1}{4Mk} \qquad \log A = \log A_1 - \alpha_2 + \frac{1}{4Mk}$$

$$\alpha_2 = \alpha + \frac{1}{2Mk} = M\alpha_1 \qquad \alpha_1 = \alpha_3 + \frac{1}{2\kappa} \qquad \kappa = M^2 k .$$

The various frequency constants average grain size dispersion, σ, and skewness χ, can readily be derived since the total integrals are known or can be obtained by simple application of conversion formulas from the integral of Form $3b°$, which is

$$\int_{x_0}^{\infty} ydx = \frac{A_1 \sqrt{\pi}}{M \sqrt{k}} \qquad N = \frac{\sqrt{\pi}}{M \sqrt{k \cdot \Delta x}} A_1 .$$

For the purpose of applying the least square method, it is best to use one of forms 4. Linear equations of condition are obtained

† Curve is published in *Phot. J.*, **49**, 134 (March, 1925).

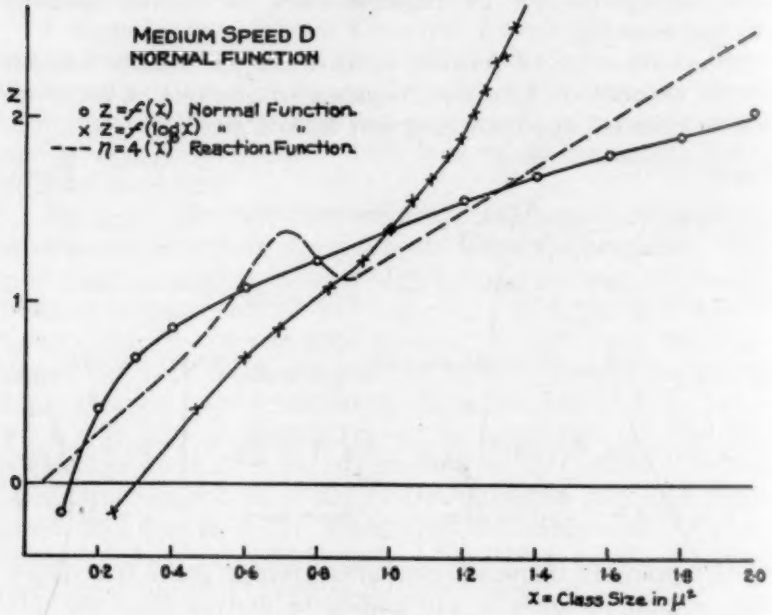
‡ The notation is consistent. ° represents generalization by introduction of x_0. $ln x = \log_e x$. $M = \log_{10} e = 0.434294$.

by application of Taylor's series, using only the linear terms. For form $4b^{\circ}$

$$\partial y = \frac{y}{A}\partial A - y[\log(x - x_0) - \alpha]^2\partial k + 2ky[\log(x - x_0) - \alpha]$$
$$\partial\alpha + y \cdot \frac{2Mk[\log(x - x_0) - \alpha]}{x - x_0}\partial x_0 \qquad (12)$$

$\partial y = o - y = F$ where o is the observed and y the calculated frequencies, and ∂A, ∂k, $\partial\alpha$ and ∂x_0 are the parameter corrections

FIG. 4.



to be added to the assumed values. By symbolizing the constants of the method the equations become:

$$F_i = X_i\partial A + Y_i\partial k + Z_i\partial\alpha + W_i\partial x_0 \qquad (12a)$$

$s = F + X + Y + Z + W$ is the checking equation. If the restricted form without x_0 is used, the last term disappears, while with natural logarithms, the factor M is dropped.

Hopes that a general frequency formula, sufficient for practical purposes, had been discovered were disappointed, however, when other photographic emulsions were studied. In Medium

Speed D, particularly, the relationship is neither regular nor continuous. Moreover, in applying the least squares method it was discovered that the differential series with respect to x_0 was only very slowly convergent, in fact, was divergent at $x = x_0$, so that unless the first assumption of the values of x_0 was very close to "correct" the whole adjustment of the parameters failed. The value of x_0 as $x = x_0$ is very critical in determining the nature of the first portion of the frequency curve. Dropping all adjustment of x_0, the remaining Gaussian normals will solve for the adjustment of formula $4b$ readily enough.

SEARCH FOR AN ADEQUATE FORMULA THROUGH NORMAL FUNCTION.

Study showed that practically all emulsions and precipitations contained discontinuities.§ While it is obviously impossible to accurately express a series of discontinuous segments by any one simple formula, a closed formula is so much the most desirable that an attempt was made to discover an adequate general formula with the aid of the normal function, assuming it to be curved and continuous for practical purposes.

The value of x_0 obtained by the graphical method of straightening the semi-logged normal function is not in general accurate enough for its *adjustment* by least squares which is so critical. However, this graphical value may be taken and the other parameters adjusted. For $\alpha 5C$ precipitations the results were as shown in Table II. For some purposes this is quite good enough. However, the values of the other parameters are so sensitive to that of x_0 that the inaccuracy in the determination of the negative x_0 greatly reduces their value as quantitative expressions of the distribution. Therefore this method, in general, was not considered satisfactory.

TABLE II.

$\alpha 5C$ Size-frequency. Formula No. $4b^6$.

$A = 3106.89$. $\alpha = 0.811708$. $k = 8.038986$. $x_0 = -0.2\ \mu^3$.

x in μ^3	0.05	0.15	0.25	0.35	0.45	0.55	0.65	0.75	0.85	0.95	
o = obser. frequency	2051.3	1130.6	510.4	302.9	130.0	66.8	32.2	9.55	3.58	3.58	
y calc.	2056.6	1124.5	547.8	268.9	135.5	70.4	37.8	20.85	11.84	6.89	
$v = y - o$	5.3	−6.1	37.4	−34.0	−5.5	3.6	5.6	11.30	8.26	3.31	$\sqrt{\Sigma v^2} = 53$

Other formulas were studied, including Rietz's $z = kx^n$ (p. 199) and a number suggested by Van Uven,[25, 26] which should be par-

§ See Part II of this paper, to be published in a future issue.

ticularly useful for this type of curve. But the discontinuities are too definite and large for one formula to fit a whole distribution with any accuracy.

III. *Representation of Data by Series.*—The Swedish astronomer, Charlier,[29] proved that all frequency distributions could be expressed by one of two mathematical series. His type A has become famous and widely used, and should, by theory, be generally applicable, while his type B is not a particularly helpful form. Type A is:

$$F(z) = N\left[\vartheta_0(z) - \frac{C_3}{3!}\vartheta_3(z) + \frac{C_4}{4!}\vartheta_4(z) - \frac{C_5}{5!}\vartheta_5(z) + - +\right] \qquad \text{Form 5.}$$

where

$$\vartheta_0(z) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{(z-\alpha)^2}{2\sigma^2}}$$

and $\vartheta_i(z)$ are the successive derivatives, while c_1, c_3, c_4, etc., are functions of the moments of $F(z)$. $\vartheta_0(z)$, the so-called generating function, contains two parameters which are determined so as to make the first two derivatives vanish. This form becomes impracticable for markedly skew distributions because of the number of terms required. H. C. Carver met this situation [31] by mathematically proving that any frequency function $\theta_0(x)$ could be substituted for $\vartheta_0(z)$. Many such have been suggested.[32, 40]

The most interesting development from our standpoint is that of Jϕrgensen,[30] ¶ who investigated the logarithmic transformation of the Gaussian form. The transformed series is represented by

$$y = F(x) = k_0\phi_0(x) - \frac{k_3}{3!}\phi_3(x) + \frac{k_4}{4!}\phi_4(x) - + - + \qquad \text{Form 6.}$$

where the generating function is

$$\phi_0(x) = \frac{1}{n\sqrt{2\pi}}\, e^{-\frac{1}{2}\frac{(\ln x - m)^2}{n}}.$$

He then shows that the parameters of the generating function in terms of the moments are

$$e^{n^2} = \frac{M_0 M_2}{M^2_1},\quad e^{2m} = \frac{M_3}{M^5_0 M^3_2},\quad k_0 = \frac{M^3_0 M_2}{M^3_1}.$$

¶ Entirely reproduced by A. Fisher, "Math. Theory of Prob.," Vol. 1. p. 236.

Although the successive derivatives of this function become rather complicated and no tables of them are published, the application of this series becomes quite practical through the transformation of the variable, so that tables of the derivatives of the error function may be used. Jørgensen gives[30] most complete tables of the latter to seven places, including the sixth derivative.

The method of moments is not well suited for the computation of the coefficients of the series since no more than four parameters can usually be determined this way with any precision unless the number of class sizes amply represented is very great. Considering the series as a finite number of terms and writing as

$$y = F(x) = k_0\theta_0(x) + k_3\theta_3(x) + k_4\theta_4(x) + k_5\theta_5(x) + +$$

Form 6*a*.

where $\theta_0(x)$ is any frequency function, it is well adapted to the method of least squares since the coefficients k are already in linear form. The generating function $\theta_0(x)$ should be one that, in itself, fits the distribution as well as possible if the parameters are to be considered significant as measures of the distribution.

Individuals accustomed to the enclosed system of frequency formulas often object to the fact that a series sometimes exhibits small negative frequencies toward the extreme end of the distribution, due of course to the limited number of terms used. This is disconcerting but cannot be said to be practically harmful since it occurs only in the range where the increasing probable error nearly or actually equals the small frequencies.

APPLICATION OF SERIES TO DATA.

This system was tested by application to 5*C* precipitation, using the Jørgensen logarithmically transformed series as seemed most appropriate from our previous study.

The parameters of the generating function may be determined by either (1) moments, or (2) least squares. Each was used for comparison, taking the extrapolated value from the double log graph in the second case also, although this need not usually be done. Jørgensen's formulas (p. 208) were used for the first method. For application of least squares it is easier to use formula 4*a*, transforming the parameters by the relations:

$$k_0 = A\sqrt{\frac{\pi}{\kappa}},\ n^2 = \frac{1}{2\kappa},\ m = \alpha.$$

Application of Taylor's series gives for the linear terms

$$\partial y = \frac{y}{A}\partial A - y\,(lnx - \alpha)^2\,\partial k + 2ky\,(lnx - \alpha)\,\partial\alpha\,. \quad (13)$$

Table III gives a comparison of the results with three terms.

TABLE III.

x in μ^3.	y Observed.	y_c Moments.	$v_m = y_c - y_o$.	y_c Least Squares.	$v_l = y_c - y_o$.
0.05	2050.0	2539.5	489.5	2048.7	− 1.3
0.15	1130.6	1370.5	239.9	1135.0	4.4
0.25	510.4	509.2	− 1.2	518.8	8.4
0.35	302.9	207.9	− 95.0	259.7	− 43.2
0.45	130.0	93.8	− 36.2	141.4	11.4
0.55	66.8	46.0	− 20.8	82.3	15.5
0.65	32.2	24.1	− 8.1	50.5	18.3
0.75	9.55	13.3	3.75	32.29	22.74
0.85	3.58	7.73	4.15	21.41	17.83
0.95	3.58	4.65	1.07	14.62	11.04

$v = y_c - y_o$ $\qquad \sqrt{v_m^2} = 555.0$ $\qquad \sqrt{v_l^2} = 60.2$

The question as to the number of terms that should be used is a difficult one since there is no quick practical test to predetermine the rapidity of convergence of a series. As the simplest test both three and five terms were independently applied by least squares. The technique is fully described by Fisher.[33] By the transformation $t = \dfrac{lnx - m}{n}$ it is possible to use the usual probability tables, such as the excellent ones of Jørgensen.

The following results were obtained:

$$\phi_0(x) = \frac{1}{n\sqrt{2\pi}}\,e^{-\frac{(lnx-m)^2}{n}} = \vartheta_0(t) = \frac{1}{\sqrt{2\pi}}\,e^{-\frac{t^2}{2}}.$$

Parameters of G. F., $\phi_0(x)$ Determined by the Method of Moments.

$m = -2.774605$ $\qquad n = 0.764531$

1. $\Omega(x) = \vartheta(t) - 5687.38\ \phi_0(x) - 477.94\ \phi_3(x) - 200.89\ \phi_4(x)$.
2. $\Omega(x) = \vartheta(t) - 5724.51\ \phi_0(x) - 593.44\ \phi_3(x) - 537.00\ \phi_4(x) - 156.33\ \phi_5(x) - 70.91\ \phi_6(x)$.

Parameters of G. F., $\phi_5(x)$ Determined by Method of Least Squares.

$m = -2.881147$ $n = 0.899309$

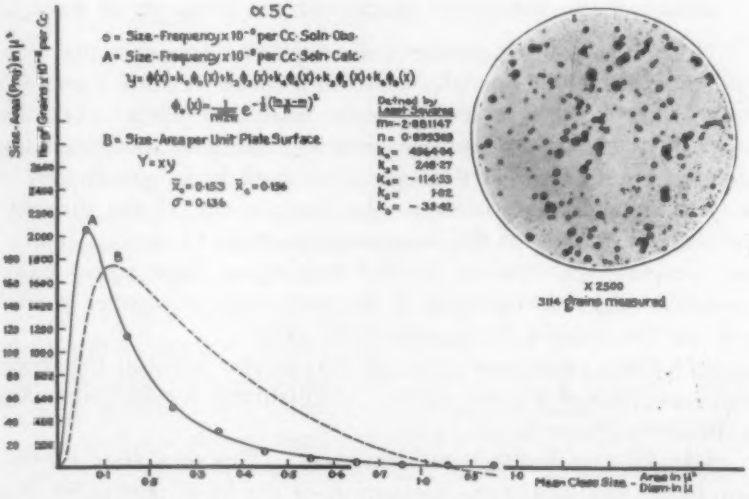
3. $\Omega(x) = \vartheta(t) = 5165.90\ \phi_0(x) + 6.426\ \phi_3(x) - 8.14\ \phi_4(x)$.

4. $\Omega(x) = \vartheta(t) = 4964.04\ \phi_0(x) + 248.27\ \phi_3(x) - 114.53\ \phi_4(x) + 1.02\ \phi_5(x) - 32.42\ \phi_6(x)$.

TABLE IV.

			Equation 1.		Equation 2.		Equation 3.		Equation 4.	
i	x in μ^2.	$\Omega(x)$	$\Omega'(x)$	v	$\Omega(x)$	v	$\Omega(x)$	v	$\Omega(x)$	v
1.	0.06	2051.3	2056.1	4.8	2051.4	0.2	2046.0	−5.3	2051.6	+0.3
2.	0.15	1130.6	1115.1	−15.5	1129.3	−1.4	1140.3	9.7	1127.9	−2.7
3.	0.25	510.4	547.2	+36.8	519.2	8.8	522.8	12.4	524.7	+14.3
4.	0.35	302.9	258.9	−44.0	278.8	−24.2	260.4	−42.6	272.3	−30.6
5.	0.45	130.0	123.2	−6.8	148.2	18.2	141.1	11.1	145.4	+15.4
6.	0.55	66.8	59.6	−7.2	74.4	7.6	80.4	13.6	75.4	8.6
7.	0.65	32.2	29.2	−3.0	33.6	1.4	48.5	16.3	35.6	3.4
8.	0.75	9.55	14.34	+4.79	11.83	2.28	30.48	20.93	12.14	2.59
9.	0.85	3.58	6.93	3.35	−4.73	−8.31	19.80	16.22	−1.11	−4.69
10.	0.95	3.58	3.18	−0.40	−4.67	−8.25	13.24	9.66	−8.45	−12.03
$\Sigma\sqrt{v^2}$				60.8		34.6		54.9		40.6

FIG. 5.


Case 5C
o = Size-Frequency x 10⁻⁶ per Cc. Soln. Obs.
A = Size-Frequency x 10⁻⁶ per Cc. Soln. Calc.
B = Size-Area per Unit Plate Surface
Y = xy
Defined by Least Squares
m = −2.881147
n = 0.899309
k₀ = 4964.04
k₃ = 248.27
k₄ = −114.53
k₅ = 1.02
k₆ = −32.42
X 2500
3114 grains measured
Mean Class Size - Area in μ² Diam. in μ
A
B

In comparing the results of the two methods of application it must be remembered that the numerical values of the derivatives attained in the tables increase enormously with each succeeding degree of differentiation, so that with appreciable values for the

coefficients, the final summated frequency becomes a residual obtained by the consecutive cancellations of very much larger numbers, a most undesirable condition. Therefore although the final fit seems, at first, to be almost equally good in each case, the much larger parameters obtained by the use of moments cause this fit to mean little except for the simple purpose of graduation. The maximum of the two curves is different, lying on opposite side of the midpoint of the first class. Moreover, the double log graph of the curve computed by least squares is smoother and more like the parabola drawn through the points.

Application by least squares of only three terms seems insufficient since the root mean square of the error is little changed and the " tail " is still badly fitted. Moreover, it is evident that terms cannot be successfully added until a fit is obtained, because later terms influence the adjustment of preceding ones. The same number of terms must be used throughout if comparisons are to be made. Therefore for precipitations and photographic emulsions five terms will be adopted, and all of the constants of the series will be determined by least squares.

SUMMARY AND METHOD OF PROCEDURE FOR ANALYSIS OF DATA.

Given the data from the size-frequency determination of a system of dispersed particles or from a series of such determinations, what is the best procedure for analytical study? Our real purpose is usually to discover general fundamental relationships connected with the distribution or involved in its production, or to give explicit expression to the comparison of the dispersed materials. In spite of the immense advantage of having data for the complete distribution divided into equal class sizes, almost invariably the determination at the extremity of smaller particle size will be limited by experimental error.

(1) Data must usually be put into tabular form of the proper units, as " frequency per 1000." Adjustment for irregular class width must be made (p. 195).

(2) Plot on double logarithm paper. This removes the inflection point and simplifies observation of the most reasonable continuous curve representative of the sample. If the size-frequency curve of the data is to be drawn by eye, it is often advantageous to read back values from the double log graph, especially for drawing in the mode. If the observed distribution is truncated

at the lower terminal the smooth curve should be projected at this end.

(3) Summate the frequencies obtaining the p values where

$$p_1 = \frac{y_1}{N}, \; p_2 = \frac{1}{N} \cdot (y_1 + y_2), \; p_i = \frac{1}{N} \cdot (y_1 + y_2 + y_3 + y_i).$$

Plot these on probability paper, both arithmetic and semi-log, or calculate the normal function (pp. 202 and 204) and plot it. The *complete distribution* must be used, although it may be necessary to obtain values of the first (smallest) class size by extrapolation on the double log graph.

(4) Determine the type of formula best representing the normal function. If it is non-continuous (contains "breaks"), it may be well to analyze the segments, particularly the lowest. Determine the constants of the normal function if an explicit formula can be assigned. It is the belief of the writers that the logarithmic form (9) (p. 205) will be found to be uniquely involved in the case of precipitates, although discontinuities may prevent its satisfactory application.

(5) Calculate the fundamental frequency constants of the observed data, total number, N, average particle size $\bar{x}$, and dispersion σ.‖ If a series of distributions is being studied these constants should be plotted against the factor being studied. For precipitations, at least, double log paper should also be used.

This may be all that is necessary for the purpose. Further analysis, consisting of graduation of the data and application of a frequency formula involves, necessarily, a proportionately very much greater period of time.

(6) If it is decided to apply a frequency formula to the data the normal function should be examined for choice of the type. For purposes of comparison, the same formula must be applied to each member of a series, even though it be possible to apply a simpler but different type to individual distributions. A closed formula is, of course, preferable to a series. If the normal function shows no discontinuities it should be possible to fit the distribution with such.

If the normal function, or graph on probability paper suggests

‖ For formulas for their calculation see any handbook such as No. 40 in Bibliography.

a formula, use it for the frequency curve. If $z = f(x)$ represents the normal function the frequency curve formula is

$$y = \frac{f'(x)}{\sqrt{2\pi}} e^{-\frac{1}{2}[f(x)]^2}.$$

Thus, if the normal function becomes a straight line on semi-log paper (or the summated frequencies on semi-log probability paper), it can be expressed by $z = \lambda \log x/x_m$ and the frequency formula can be expressed as either form 3 or 4 (p. 205), which differ only in the parameters.

If the distribution is not completely known or is markedly skew, the formula is probably better applied by least squares rather than by moments, putting the parameter adjustments into linear form by application of Taylor's series (p. 206).

(7) If the single closed formula does not give a satisfactory fit, due to discontinuities or otherwise, use it as the generating function of a translated Charlier series (p. 209). This can usually be decided *a priori* from the appearance of the normal function graph. Make transformation of variable $z = f(x)$, which is the normal function, and use tables of the error function and its derivatives (p. 210). For inter-comparison the number of terms used in the series must be the same throughout a set of distributions. The coefficients of the terms should be determined by least squares.

If the normal function suggests no explicit expression other generating functions of general type, as suggested by various individuals,[32, 34] may be used.

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The Heat Energy of X-rays. R. KEGERREIS. (*Phys. Rev.*, June, 1927.)—A beam of X-rays was sent into a deep cylindrical lead cup in which were eventually absorbed all the reflected, the scattered and the fluorescent X-rays along with any beta-rays produced. The lead vessel was surrounded by four copper cylinders and by both oil and water, the whole forming a complex calorimeter. The X-ray tube had a tungsten target and was operated by a Snook Special X-ray machine giving a pulsating direct current of 120 pulses a second. The energy delivered to the tube was found by immersing it in a vessel of oil and measuring the rate of rise of temperature. This assumes that all the energy furnished to the tube finally is converted into heat, none of which lies outside of the vessel. With a peak voltage of 98.5 kilovolts, the input was 270 watts, of which only .18 per cent. was converted into the power of the emitted X-ray beam. With a peak voltage of 196.3 kv. the corresponding figures are 499 watts and .28 per cent. The efficiency is roughly proportional to the peak voltage, while the X-ray output is about in proportion to the square of this voltage. G. F. S.

Uncommon Manganese Deposit. (U. S. Department of the Interior Press Memorandum.)—The presence of manganese minerals on the eastern and southern borders of the Olympic Mountains, Washington, was known as early as 1912, when they were recognized in some copper prospects, but their possible value as a source of manganese only became evident during the World War. Attempts were made to work the deposits but were unsuccessful because the only materials that could be found contained too much silica to permit their conversion into alloys of acceptable grade. These deposits, which are known to be large, may yet become an important resource if the silica can be eliminated.

Since the war, however, prospecting along the north slope of the mountains has led to the discovery of several deposits, one of which near Lake Crescent has yielded in three years about 17,000 tons of ore containing more than 52 per cent. of manganese and less than 9 per cent. of silica, probably the largest concentrated body of this grade that has yet been found in the United States. Further prospecting has shown that there are other deposits on the southwest side of the mountains. Bulletin 795-A—"Manganese-bearing Deposits near Lake Crescent and Humptulips, Washington"—a report of a preliminary study of all these deposits by a geologist of the Geological Survey—shows that although the mineral composition varies from place to place, they all lie at nearly the same stratigraphic position in the rock section, and the hope is held out that more deposits of high-grade ore may be discovered in the same belt. Prospecting in the region is much hampered by the heavy forest and thick cover of underbrush and moss, but as the timber is cut the search will be less difficult. The principal deposit was discovered in an area swept by forest fires several years ago.

SPECTRAL INTENSITY DISTRIBUTION IN A HYDROGEN DISCHARGE.

BY

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ABSTRACT.

A COLD cathode discharge tube has an auxiliary tube attached from which cathode-rays are projected against the main cathode. A photo-electric cell, attached to a monochromatic illuminator, is used to measure relative intensity distribution of H_γ and H_δ respectively, from the main cathode through the cathode dark space into the negative glow with and without excitation of the auxiliary tube. When the main cathode is bombarded by the electron stream the intensity of spectral illumination in the negative glow is increased by about 20 per cent. This increase does not result from the mere addition of an illumination, which appears when the auxiliary tube is alone excited, to the illumination of the main discharge, but may be attributed to the production of soft X-rays in the gas which are capable of exciting the gas molecules.

R. Seeliger and co-workers [1] have investigated spectrophotometrically the several characteristic sections of a cold cathode discharge tube. Their method consisted of an examination, with a microphotometer, of spectrograms taken at points along the discharge. The intensity distribution of any spectral line was found continuous in passing from one portion of the discharge to another, *e.g.*, from the Faraday dark space into the positive column, and that the maxima of illumination for different lines appeared displaced relative to one another.

A. Wehnelt and A. Jachan [2] demonstrated with several experimental arrangements the effect of bombarding the cold cathode of an ordinary discharge tube with a beam of cathode-rays. There resulted an increase in the *total* intensity of illumination in the tube together with a shrinkage of the cathode dark space.

[1] R. Seeliger, *Ann. d. Phys.*, **67**, 352, 1922. R. Seeliger and J. Okubo, *Phys. Zeit.*, **25**, 337, 1924. R. Seeliger and W. Lindow, *Phys. Zeit.*, **26**, 393, 1925.

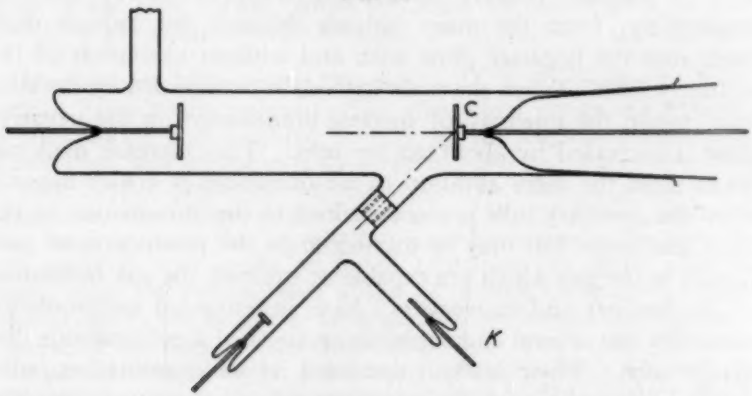
[2] Wehnelt and Jachan, *Zeit. f. Phys.*, **31**, 666, 1925.

The purpose of the present investigation was to examine the effect such an electronic bombardment would produce upon the *spectral* intensity distribution near the cathode of a hydrogen discharge tube. Because of experimental limitations, the changes in H_γ and H_δ alone were capable of investigation.

EXPERIMENTAL ARRANGEMENT.

The discharge tube employed (Fig. 1) in these investigations was similar to one of those employed by Wehnelt and Jachan. The aluminum electrodes in the main tube were 2 cm. in diameter and had a separation of 10 cm. The ground-glass joint permitted removal of the main cathode to clean the tube. The

FIG. 1.



aluminum diaphragm in the auxiliary tube had a length of 1 cm. and was perforated with five small holes in order to permit the delivery of a parallel bundle of cathode-rays by the auxiliary tube. The diameter of the main tube was 3 cm. and that of the auxiliary tube 1 cm.

Small lead accumulators with a total voltage of 800 were used to excite the main discharge, while a 30,000-volt transformer excited the auxiliary discharge. In the arrangement of Wehnelt and Jachan, the cathode C and the electrode K were electrically joined. In the present arrangement, this connection had to be eliminated; otherwise, excitation appeared throughout the entire system.

A ballast of about five litres capacity assured nearly a constant pressure which was measured on a McCleod gauge. Hydro-

gen, generated electrolytically, dried by passing over calcium chloride and phosphorus pentoxide, and freed from traces of oxygen by the heated filament of a 1000-watt tungsten lamp, was supplied to the system.

Measurements of relative intensity distribution of the hydrogen lines were made with a Kunz type photo-electric cell equipped with a guard ring. A spectrometer was mounted on a carriage which could be moved by a simple screw arrangement over a pair of rails laid parallel to the discharge tube. The eye-piece of the spectrometer telescope was replaced by a vertical slit placed in the focal plane of the objective. The photo-electric cell was attached to the telescope. With the cell fixed in any one position, relative to the telescope, the same photo-active surface was illuminated as the spectrometer was moved along the discharge tube. A second slit attached in front of the usual collimator slit permitted of examining a thin cross-section of the discharge at any time.

A Dolezalek electrometer indicated relative photo-electric currents. A suitable India ink leak shunted the two pairs of quadrants and converted the electrometer into a steady deflection instrument. Stray electric fields and body capacity effects were eliminated by moving the spectroscope within a grounded metal box with two windows, one for electrometer observations and the other to receive light from the discharge tube. The interior of this box was painted a dull black.

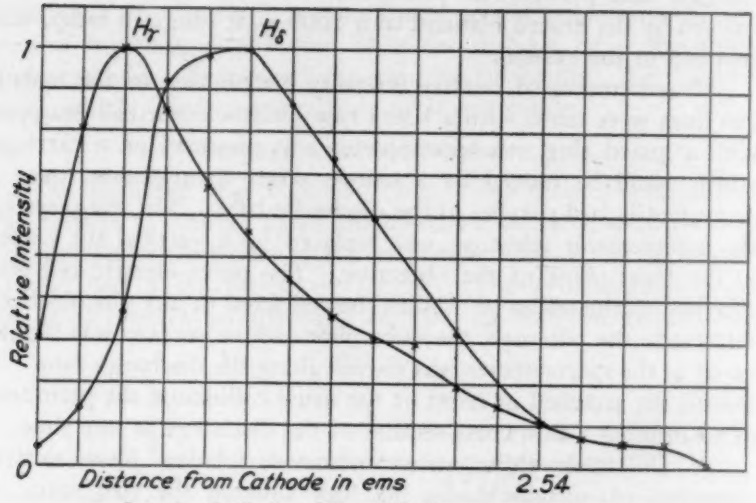
Relative humidity of the atmosphere affected the "dark" current of the photo-electric cell to such an extent that a controllable amount of stray light had to be admitted to the guard box to reduce this current to zero.

OBSERVATIONS.

The relative intensity distributions of H_γ and H_δ in the negative glow, when the total discharge current was 33 m.a. and the gas pressure 2 mm., are given in Fig. 2. These results are comparable with those of R. Seeliger and co-workers who examined spectrograms with a microphotometer. The intensity of H_β was too feeble to produce any photo-electric current.

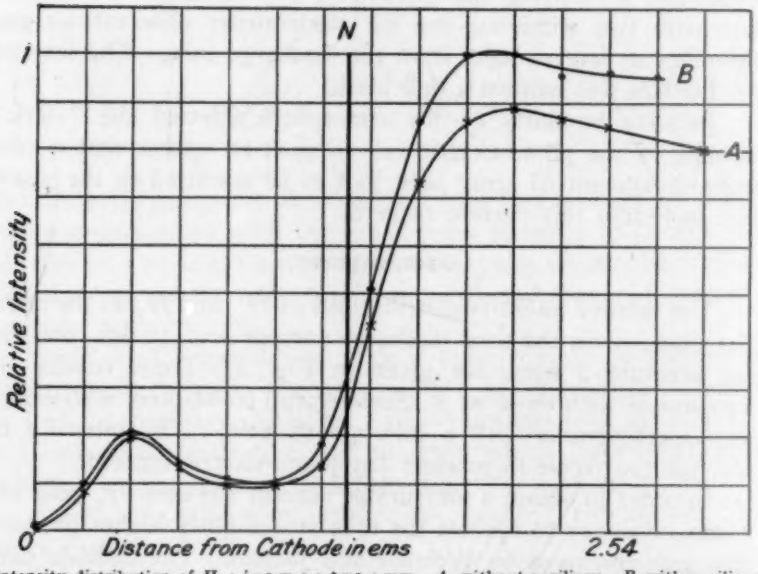
In order to obtain a measurable current for even H_γ and H_δ, it was necessary to operate the tube at a slightly higher pressure than was employed by Wehnelt and Jachan. The cathode dark space continued to show a shrinkage when the main cathode was

FIG. 2.



Intensity distribution of H_γ and H_δ; $i = 33.0$ m.a.; $p = 2.0$ mm.

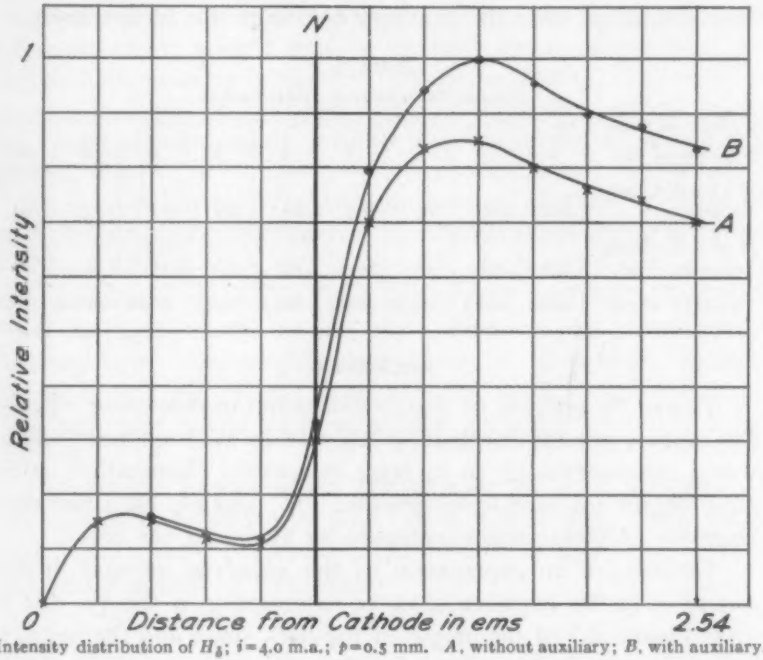
FIG. 3.



Intensity distribution of H_γ; $i = 2$ m.a.; $p = 0.4$ mm. *A*, without auxiliary. *B*, with auxiliary.

bombarded by the electron stream at this pressure. Fig. 3 shows the increase in intensity of H_γ when the main cathode was bombarded by the high-speed electron stream from the auxiliary tube. The current in this case was 2.0 m.a. and the gas pressure 0.4 mm. Similar curves are shown for H_δ in Fig. 4 which were obtained for a current of 4.0 m.a. and a pressure of 0.5 mm. In each of the last two figures, the boundary between the cathode dark space and the negative glow, as determined by the unaided

FIG. 4.



Intensity distribution of H_δ; i=4.0 m.a.; p=0.5 mm. *A*, without auxiliary; *B*, with auxiliary.

eye, is represented by the heavy vertical line N. Curve A represents relative intensity distribution in the main discharge alone; curve B, that for the main and auxiliary discharges combined. The small humps in these curves, lying between the boundary N and the cathode C, identify the cathode glow. The maximum increases of intensity in the negative glow proved to be of the order of 20 per cent. in both cases.

When the auxiliary discharge was operated alone, the main tube was filled with a faint glow which was most intense near the

cathode, the region investigated. In order to ascertain whether or not the curves B were a result of the superposition of this glow upon the illumination of the main discharge, observations were made of H_γ with only the auxiliary discharge in operation. In Table I are tabulated relative intensities produced by the main discharge, the auxiliary discharge plus the main discharge, and the auxiliary discharge alone. It is evident from the examination of this table that no superposition of intensities could account for the increased illumination of H_γ (and likewise H_δ) in the main discharge when the auxiliary discharge was in operation.

TABLE I.

Relative Intensities of Illumination.

Position of slit ..	0	1	2	3	4	5	6	8	10
Main discharge alone	.006	.08	.19	.13	.10	.09	.13	.73	.89
Main discharge plus auxiliary..	.009	.10	.21	.14	.11	.10	.18	.86	1.00
Auxiliary alone..	.0028	.0025	.0023	.0025	.0023	.0023	.0020	.0020	.0020

DISCUSSION.

It was the purpose of this investigation to determine whether the increase in total illumination reported by Wehnelt and Jachan[2] was a consequence of an increase in spectral illumination to the same degree for each hydrogen line. H_γ and H_δ each had their intensity of illumination increased by about 20 per cent.

Ground for an explanation of this observed increase in illumination in the negative glow is contained in a theory, due to J. J. Thomson,[3] of the origin of the dark space and the negative glow in a discharge tube. Briefly, this theory is as follows: Three types of radiation exist in the vicinity of the cathode. First, the neutralization of positive ions at the cathode produces a radiation which is capable of emitting photo-electrons from the cathode. Second, these photo-electrons are accelerated by the electric field in the dark space which results in the emission of a soft radiation by collision with gas molecules. Third, this soft radiation ionizes the gas molecules and produces the negative

[3] J. J. Thomson, *Phil. Mag.*, **48**, 1, 1924.

glow. The existence of the soft radiation in a discharge has been demonstrated by J. J. Thomson.[4]

It is possible, in the present experiments, that the cathode stream from the auxiliary tube produces a soft radiation in a manner like Thomson's photo-electrons which results in the excitation of molecules in addition to the normal excitation. This is further brought into evidence by the shrinking of the dark space for the electrons from the auxiliary tube are capable of producing the soft radiation in all directions, whereas Thomson's photo-electrons, being directed away from the cathode under the influence of the electric field in the dark space, would emit the soft radiation mainly in the direction of the anode. The electrons emanating from the auxiliary tube, not being of a critical velocity, could simultaneously enhance the brightness of all of the spectral lines.

It was shown by Wehnelt and Jachan [2] that the increase in total illumination could likewise be effected when the axis of the auxiliary tube was normal to the axis of the main tube so that the cathode-rays passed in front of the main cathode. This fact lends strength to the above explanation of the observations reported here. One might otherwise be led to suppose, through the design of the present tube, that the cathode-rays from the auxiliary tube would emit secondary electrons from the main cathode.

The writer expresses his gratitude to the staff members of the Research Laboratory, Nela Park, Cleveland, where these experiments were performed, for valuable suggestions offered during the course of this work.

[4] J. J. Thomson, *Phil. Mag.*, **49**, 761, 1925.

Increase in Electricity Used. (U. S. Department of the Interior Press Memorandum.)—The monthly report of the production of electricity by public-utility power plants for May, issued by the Geological Survey, shows a total for the first five months of the year amounting to 32.4 billion kilowatt-hours, or about 9½ per cent. more than the output for the same period in 1926. An estimate based on the output so far this year and on comparable figures for 1926 indicates that the total output for 1927 will be in the neighborhood of 80 billion kilowatt-hours. The total for 1926 was 73.8 billion kilowatt-hours.

The effect of the increasing hours of daylight and the rising temperature from the first to the middle of the year on the demand for electricity for light and heat is shown in the decrease in the average daily production of electricity that occurs during this period each year. This year the decrease in the average daily output from January to May was about 3 per cent.; in 1926 the decrease was about 5 per cent.; in 1925, 6 per cent. The lessening of the decrease in 1927 may be due, in part, to the increasing load for electrical refrigeration.

The average daily production of electricity by the use of water power in 1927 has set a record for each month from March to May. As the amount of electricity produced by water power is directly dependent on the amount of water flowing in the streams used for the development of power, it is expected that the output will begin to decrease after the month of May, as March, April, and May are generally months of maximum flow for streams in the United States.

Notwithstanding the record-breaking output of petroleum this year, the consumption of fuel oil by public-utility power plants has decreased each month, and the consumption in May was less than in any other month since the Geological Survey began to publish monthly power reports in 1919.

The Rigidity of Solid Unimolecular Films. H. MOUQUIN and E. K. RIDEAL. (*Proc. Roy. Soc.*, A769.)—When benzene solutions of palmitic or stearic acid placed upon water surfaces are let evaporate at low temperatures there remains in each case a sheet of oriented solid acid one molecule thick. Such films are exceptionally strong. This paper presents a method of measuring their rigidity. A disc parallel to the surface of the film and 1.5 mm. beneath it in the water was rotated. Through the viscosity of the water a torque was exerted on the film and its distortion was found by measuring the displacement of dust particles on the film, the latter being firmly attached at its periphery to the inner surface of the round dish containing the water. The coefficients of rigidity are the same for stearic and palmitic acid, the common value being 4.9×10^7 dynes per cm.^{2}. For iron the corresponding value is 7.6×10^{11} and for rubber 1.6×10^7. G. F. S.

HYSTERESIS IN VACUUM TUBE OSCILLATORS.*

BY

LAURISTON S. TAYLOR, A.B.

Department of Physics, Cornell University.

IN AN earlier investigation,[1] it was shown how, when a blocking condenser and resistance were placed in the grid circuit of a vacuum tube oscillator, the oscillations were broken up into groups which for convenience are called "zules." It was pointed out that such a circuit was capable of operation in two states, namely, one of continuous and one of intermittent oscillation. The actual state of oscillation is determined chiefly by the constants of the oscillatory circuit in which oscillation is maintained by the vacuum tube. Thus for any given value of the grid condenser and by-pass, the two states can be produced by changing the inductance or capacity in the oscillatory circuit. An empirical relation was obtained for the zule frequency F in terms of these constants. Thus

$$F = A\varepsilon^{\frac{L_2 - kC_2}{2L_1}},$$

where A and k are constants and L_2, L_1, and C_2 are in the oscillatory circuit.

The constants of that part of the circuit determine the rate of accumulation of charge on the grid and consequently the state of depression of the grid potential. The grid depression was then shown to be one of the two major factors in determining the state of oscillation, the other being a type of oscillation hysteresis.

At the start of a zule, the mean grid potential was found to be above the dynamic grid potential cut-off. In this study a more precise location of the mean E_g was made. At the start of the zule, E_g was slightly higher than the static cut-off and at the end of the zule was slightly lower than the dynamic cut-off. Thus the actual range of variation of the mean E_g is determined by these two cut-offs, the first of which remains fixed for any given tube.

* Submitted for publication May 2, 1927.

[1] L. S. Taylor, JOUR. FRANK. INST., **203**, p. 351, Mar., 1927.

There was subsequently made available a synchronized cathode-ray oscilloscope with a linear time axis,[2] by means of which the theory could be checked. Two assumptions had been made: (1) That the circuit when oscillating in the intermittent state did not suffer any appreciable wave distortion within the zule. (2) That the oscillation frequency within the zule remained constant. When the wave was observed by means of the oscilloscope, both these assumptions were found to be true. Fig. 1 is a copy of the oscilloscope pattern showing the wave form in the oscillatory circuit. Synchronization was adjusted for the zule frequency and not the oscillation frequency, and so the pattern shows directly the envelope of the oscillations. Intermediate potential variations were seen faintly in the background. Two points in particular will be noted. First, the zule builds up quickly to a maximum at the *beginning* of oscillation and then drops off more gradually as the grid depression proceeds. The amplitude falls off continuously to zero. Secondly, between zules there is no oscillation nor irregularity of any kind.

In a paper by Appleton, Watt and Herd,[3] similar to the one mentioned above [2] which was submitted at about the same time, the investigators stated that the oscillations were "suddenly quenched." Fig. 1 shows the actual fact to be quite the contrary as pointed out, since in the particular case the oscillations die off to zero in roughly three times the time necessary to build up. A similar misstatement was made by the author [4] regarding a Colpitts circuit and while this circuit was not actually checked by the oscilloscope, it seems reasonable to suppose that the zule dies off slowly and not suddenly.

Fig. 2 shows the envelope of the grid potential variations over a zule. The hysteresis action taking place as caused by the grid depression is shown strikingly in this figure. The envelope is no longer symmetrical, but varies about a mean which decreases in some manner with time. Oscillation ceases at a certain value N of the grid potential, as determined by the dynamic cut-off. From this point the mean increases exponentially to a higher value M which is determined by the static cut-off. During the interval

[2] Bedell and Reich, Pittsfield Meeting, A.I.E.E., May 25, 1927.

[3] Appleton, Watt and Herd, *Proc. Roy. Soc.*, A, 3, p. 615 (1926).

[4] L. S. Taylor, *J.O.S.A.R.S.I.*, 12, p. 149 (Feb., 1926).

NM the circuit is again seen to be in a state of non-oscillation. Thus the theory given before was found to be true.

There was previously some doubt as to whether the transition from one state to another was perfectly continuous. Without further adjustment of the circuit constants, it is possible to effect this transition by means of an external biasing potential applied to the grid. When observed by means of the oscilloscope during this process, it was found that the transition proceeded continuously, and that there was no sudden jump between the two states.

Distortion of the zule form was found to occur under certain conditions. By application of a strong bias to the grid, such that the zule was of comparatively long duration (several seconds), it

FIG. 1.

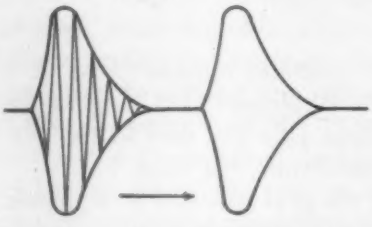
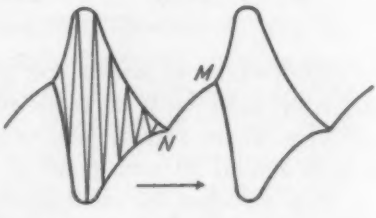


FIG. 2.



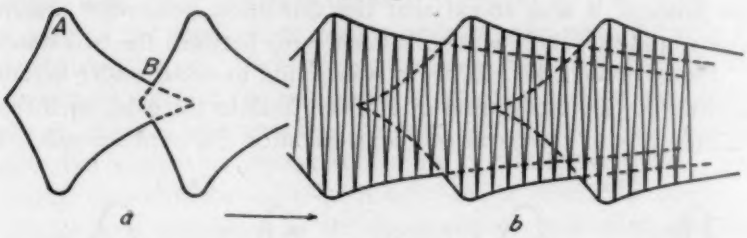
was found that the oscillations built up slowly to the maxima and then died off very rapidly. This is seen to be just the reverse of the form for the more rapid zules as shown in Fig. 1. The length of the zule was found to vary slightly with the application of an external grid bias when the zule frequency was slow (*i.e.*, less than about 20 per minute). For very low zule frequencies the change was quite marked.

Another point of uncertainty was checked. For extremely high zule frequencies it was previously thought that an entire zule occurred during a single impressed e.m.f. cycle so that the zule effect disappeared and continuous oscillation alone resulted. Actually it was found that the zules "overlapped" and the complete zule was never formed. Fig. 3*a* shows the action taking place within the oscillatory circuit. Oscillation reaches a maximum at *A* and starts to die out in the usual manner. However, before the grid potential is depressed to the cut-off point, the leakage of the trapped charge begins to exceed the rate of recharging and at *B* the next zule begins. Thus there is seen to be no

longer an intervening period of non-oscillation. Fig. 3*b* shows the form (to a different time scale) as the zule frequency is still further increased. Eventually the form is found to be but a ripple in the successive amplitudes of oscillation, and finally even the ripple disappears and all hysteresis action is absent.

From the latter two patterns it is clear that there is no state

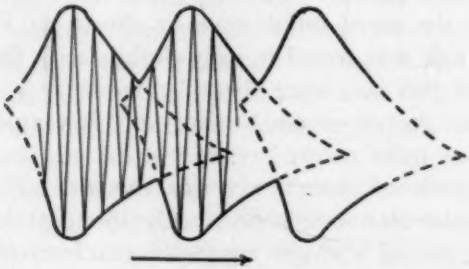
FIG. 3.



of equilibrium at which the grid potential remains *fixed* at a negative value such as reported by Appleton.[3] As long as the grid condenser and by-pass resistance play any part in the hysteresis action, no such point of equilibrium can exist.

Fig. 4 shows the oscillation of the grid potential at high zule frequencies. In general it is the same as discussed above, except

FIG. 4.



between the zule maxima. Here it will be noted that the oscillations occur about a rising mean potential similar to that shown in Fig. 2.

I am indebted to Mr. H. J. Reich, of this laboratory, for his assistance in the use of the oscilloscope which was developed by Prof. F. Bedell and himself.

ELECTRIC DOUBLE REFRACTION IN RELATION TO THE OPTICAL ANISOTROPY OF GASEOUS MOLECULES.

BY

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Communication No. 15.

THE accepted explanation of this effect, the feeble birefringence exhibited by gases and liquids when placed in an electrostatic field, is that the electrostatic field exercises an orientative influence on the molecules, that the latter possess an intrinsic optical anisotropy and that, in consequence of the molecular orientation, the gas becomes optically anisotropic. Although no serious attempts have been made to connect the intrinsic optical anisotropy with the familiar properties of the molecule disclosed in recent years through the investigation of band spectra, the general equation of Kramer giving the relation between the index of refraction and these molecular properties ought to be capable of accounting for this type of effect. From Kramer's formula for the index of refraction, n, that

$$\frac{n^2-1}{n^2+2}=\alpha\sum_s\left[\sum_r\frac{N_sA_{r\to s}\lambda^2_{rs}\lambda^2}{\lambda^2-\lambda^2_{rs}}-\sum_t\frac{N_sA_{s\to t}\lambda^2_{st}\lambda^2}{\lambda^2-\lambda^2_{st}}\right], \quad (1)$$

where N_s is the number of molecules in the state s, $A_{r\to s}$ the probability of a transition from the state r of higher energy content than s to s, $A_{s\to t}$ the converse probability, λ_{rs} the wavelength of the light absorbed in this transition and λ_{st} the wavelength emitted during the second type of transfer, that of emission, the author has been able from general statistical considerations to express in terms of the external properties of the gas as a whole, such as the temperature T, the electric intensity E of the applied field and the total number of molecules N, the molecular quantities, the A's and N's, entering into this equation; thus

$$A_{rs}=A^r{}_s(E)e^{\frac{\epsilon_r}{kT}-\frac{\mu_rE}{kT}}$$

$A^r{}_s(E)$ being a quantity which is independent of the temperature but may depend on the electric field, and

$$N_s = Ne^{-\frac{\varepsilon_s}{kT}+\frac{\mu_s E}{kT}},$$

where ε_s is the energy of the s^{th} state for vanishing electric field E, and μ_s is the electric moment of the molecule in that state.

Substituting these equalities in Kramer's relation for the index of refraction, we obtain the following equation expressing the relation between the refractive index squared, the temperature and the electric field,

$$\frac{n^2-1}{n^2+2} = \beta p \sum_s \sum_r A^r{}_s(E)\, e^{\frac{h\nu(rs)}{kT}} e^{-\frac{(\mu_r-\mu_s)E}{kT}} \frac{\lambda^2_{rs}\lambda^2}{\lambda^2-\lambda^2_{rs}}, \qquad (2)$$

where β is a constant and p the pressure of the gas.

Letting n_p and n_s be the refractive indices for light vibrating parallel and perpendicular, respectively, to the applied electric field, the specific rotation, Kerr's constant for an applied field whose electric intensity is E, is given by the relation,

$$K = \frac{n_p - n_s}{\lambda E^2}, \qquad (3)$$

where λ is the wave-length of the incident light.

Now the only quantities appearing in equation (2) which can depend on the orientation of the plane of polarization of the incident light to the direction of the applied electric field are the probability coefficients, the $A^r{}_s(E)$'s. If we, therefore, consider the case of a molecule in which the axis of the permanent moment coincides with the x-axis, then the probability coefficient for transitions along the polar axis—the x-axis—and in the equatorial plane—the yz plane—will in general be different so that the coefficients $A^r{}_s(E)$ will assume distinct values depending on the direction of transition with respect to the axis of the average moment of the gas, *i.e.*, the direction of the electric vector E.

Letting the probability of transition be $A'^r{}_s(E)$ for the case where the incident light vibrates in the plane of the electric vector E and the corresponding quantity be $A''^r{}_s(E)$ for light vibrating perpendicularly to the direction of the electric intensity, then

$$\frac{n^2_p-1}{n^2_p+2} = \beta p \sum_s \sum_r A'^r{}_s(E)\, e^{\frac{h\nu(rs)}{kT}} e^{-\frac{(\mu_r-\mu_s)E}{kT}} \frac{\lambda^2_{rs}\lambda^2}{\lambda^2-\lambda^2_{rs}}, \qquad (4)$$

with a similar equation for n_s except that $A'^r{}_s(E)$ is replaced by $A''^r{}_s(E)$, whence

$$n_p - n_s = \frac{1}{2}(n^2{}_p - n^2{}_s) = \frac{1}{2}\left\{(n^2{}_p - 1) - (n^2{}_s - 1)\right\}$$

$$= \frac{3}{2}\beta p \sum_s \sum_r \left\{A'^r{}_s(E) - A''^r{}_s(E)\right\} e^{\frac{h\nu(rs)}{kT}} e^{-\frac{(\mu_r - \mu_s)E}{kT}} \frac{\lambda^2{}_{rs}\lambda^2}{\lambda^2 - \lambda^2{}_{rs}}.$$

Expanding $e^{-\frac{(\mu_r - \mu_s)E}{kT}}$ in terms of E and retaining only the first two terms, the first of which is a scalar quantity and the second a vector, we deduce the new equation

$$n_p - n_s = \frac{3}{2}\beta p \sum_s \sum_r \left[\left\{A'^r{}_s(E) - A''^r{}_s(E)\right\} e^{\frac{h\nu(rs)}{kT}} \frac{\lambda^2{}_{rs}\lambda^2}{\lambda^2 - \lambda^2{}_{rs}}\right.$$

$$\left. - \left\{A'^r{}_s(E) - A''^r{}_s(E)\right\} e^{\frac{h\nu(rs)}{kT}} \left(\frac{\mu_r - \mu_s}{kT}\right) E \frac{\lambda^2{}_{rs}\lambda^2}{\lambda^2 - \lambda^2{}_{rs}}\right]. \quad (5)$$

The first term of this expression, which is independent of E, vanishes if the gas is not optically active, since the statistical averages of $A'^r{}_s(E)$ and $A''^r{}_s(E)$ are the same for the random distribution of polar axes which arises whenever the molecules of a gas are independent of external electrical influences; the second term becomes, on replacing $\left(\frac{\mu_r - \mu_s}{kT}\right)$ by its statistical average in the direction of E, *i.e.*, by $\frac{(\mu_r - \mu_s)^2}{3kT}E$,

$$n_p - n_s = \frac{1}{2}\beta p \sum_s \sum_r \left\{A'^r{}_s(E) - A''^r{}_s(E)\right\} \times$$

$$e^{\frac{h\nu(rs)}{kT}} \frac{(\mu_r - \mu_s)^2 E^2}{(kT)^2} \frac{\lambda^2{}_{rs}\lambda^2}{\lambda^2 - \lambda^2{}_{rs}}. \quad (5')$$

This same equation can also be derived in the following manner, which has some advantages on account of the simplicity of the various steps involved in the analysis.

The change in the refractive index n for unpolarized light due to the impressed field of electric intensity E is

$$\Delta n = \frac{3}{2}\beta p \sum_s \sum_r \overline{A}^r{}_s(E)\, e^{\frac{h\nu(rs)}{kT}} \frac{(\mu_r - \mu_s)^2}{3(kT)^2} E^2 \frac{\lambda^2{}_{rs}\lambda^2}{\lambda^2 - \lambda^2{}_{rs}},$$

where $\mu_r - \mu_s$ has been replaced by its average value in the direction of E,

$$\frac{(\mu_r - \mu_s)^2}{3kT} E$$

and where $\bar{A}^r{}_s(E)$ is the average value of $A^r{}_s(E)$ in any direction for a random distribution of polar axes.

For light vibrating in the plane of E,

$$\Delta n_p = \frac{3}{2} \beta p \sum_s \sum_r A'^r{}_s(E)\, e^{\frac{h\nu(rs)}{kT}} \frac{(\mu_r - \mu_s)^2}{3(kT)^2} \frac{\lambda^2{}_{rs}\lambda^2}{\lambda^2 - \lambda^2{}_{rs}},$$

so that

$$\frac{\Delta n_p - \Delta n_s}{E^2} = \frac{1}{2} \beta p \sum_s \sum_r \left\{ A'^r{}_s(E) - A''^r{}_s(E) \right\} e^{\frac{h\nu(rs)}{kT}} \times \frac{(\mu_r - \mu_s)^2}{(kT)^2} \frac{\lambda^2{}_{rs}\lambda^2}{\lambda^2 - \lambda^2{}_{rs}},$$

the same equation as that derived by the former method.

This quantity, $\mu_r - \mu_s$, the difference in the electric moment of the molecule in the two states r and s, respectively, we shall denote for brevity in the future by μ_{rs}. This is the same quantity which is measured by the experiments on the dielectric constant of polar gases whereas the quantity appearing in the theory of band spectra is the true moment μ_r of the molecule.

If one term of the series predominates over the remainder in absolute value, then

$$n_p - n_s = \frac{\beta}{2} p \frac{\left\{ A'^r{}_s(E) - A''^r{}_s(E) \right\}}{\bar{A}^r{}_s(E)} \bar{A}^r{}_s(E)\, e^{\frac{h\nu(rs)}{kT}} \frac{\mu^2{}_{rs}E^2}{(kT)^2} \frac{\lambda^2{}_{rs}\lambda^2}{\lambda^2 - \lambda^2{}_{rs}}$$

$$= \frac{1}{2} \left\{ \frac{A'^r{}_s(E) - A''^r{}_s(E)}{\bar{A}^r{}_s(E)} \right\} \frac{n^2 - 1}{n^2 + 2} \left(\frac{\mu_{rs}E}{kT} \right)^2;$$

hence

$$K = \frac{n_p - n_s}{\lambda E^2} = \frac{n^2 - 1}{6\lambda} \left(\frac{\mu_{rs}}{kT} \right)^2 \left\{ \frac{A'^r{}_s(E) - A''^r{}_s(E)}{\bar{A}^r{}_s(E)} \right\}. \quad (6)$$

In order to obtain an idea of the magnitude of the ratio $A''^r{}_s(E) - A'''^r{}_s(E)/\bar{A}^r{}_s(E)$ we shall have to correlate this quan-

tity with some observable phenomena for which we have accurate data, *e.g.*, the phenomenon of the scattering of polarized light in gases. The most significant fact relating to the scattering of polarized light in gases is the imperfection of the polarization of the light scattered in a direction perpendicular to the beam. When a polarized beam of light impinges on a molecule, which is in a state to absorb it, the light emitted by the molecule, *i.e.*, the scattered light, will have the same polarization as the original beam provided the probability coefficient has the same value for all directions of absorption in the molecule. Where, however, as in the case of a polar molecule, the value of the absorption probability coefficient is different in the direction of the polar axis from that in the equatorial plane, then the scattered radiation will no longer have the same direction of polarization as in the preceding case of a symmetrical molecule. The ratio of the amount of light scattered with its plane of polarization perpendicular to plane of polarization in the case of scattering from a symmetrical molecule to the total amount of light scattered will be r:

$$r = \frac{A'_s(E) - A''_s(E)}{A'_s(E) + A''_s(E)} = \frac{A'_s(E) - A''_s(E)}{2\bar{A}_s(E)},$$

where r is the factor of depolarization.

From which we finally obtain the relation

$$K = \frac{n_p - n_s}{\lambda E^2} = \frac{n - 1}{6\lambda}\left(\frac{\mu_{rs}}{kT}\right)^2 \cdot 4r. \qquad (7)$$

Here we have a simple relation connecting the Kerr constant, the depolarization constant r, the index of refraction and the electric moment μ_{rs} of the molecule, from which we should be able to deduce accurate values for the electric moment μ_{rs} in cases where the other quantities are known.

From observations on HCl we have the following data:

$k = .90 \times 10^{-10}$ per atm.

$n = 1.000447$

$r = .010$

from which, substituting into equation (6), we find for the electric moment, the value

$$\mu_{rs} = .98 \times 10^{-18} \text{ e.s.u.},$$

which agrees well with the value

$$\mu = 1.03 \times 10^{-18} \text{ e.s.u.},$$

found by Zahn from measurements on the temperature coefficient of the dielectric constant.[1]

From a consideration of the induced moments due to the applied electric and electromagnetic fields, assuming the forces of restraint depend on the direction of the displacement with respect to the axis of the permanent moment, Raman and Krishman [2] have deduced an analogous equation to (7) for the relation between the Kerr constant and the physical properties of the molecule, namely,

$$K = \frac{n-1}{5\lambda}\left(\frac{\mu}{kT}\right)^2 \sqrt{\frac{5r}{6-7r}} \qquad (8)$$

in our notation.

The difference between equations (7) and (8) arises through the difference in the fundamental concepts serving as the basis of the two distinct treatments of the same problem, the one being developed by using the theory of the virtual oscillator, the other, the classical oscillator.

As the two equations give equally satisfactory values for the electric moment of HCl, the only case where sufficient data are available to make a decision concerning the relative merits of the two equations, and as the measurements of the depolarization factor r are subject to relatively large experimental errors, no comparison of the validity of the two distinct treatments is possible.

CONCLUSION.

From a consideration of the variation of the index of refraction of a gas with electric intensity due, first, to the change in the potential energy of the molecular system arising from the reaction of the electric field on the permanent moment of the molecule, and, secondly, to the dependence of the probability of transition on the relative orientation of the direction of the electric vector in the incident wave, and therefore of the direction of transition with respect to the axis of the permanent moment, a

[1] C. T. ZAHN, *Phys. Rev.*, Oct., 1924.
[2] *Phil. Mag.*, April, 1927.

value for the Kerr constant, *i.e.*, the specific rotation of the plane of polarization in unit electric field intensity, is derived. The dependence of the probability of transition with direction relative to the polar axis explains the optical anisotropy of the molecular structure in terms of properties of the molecule which arise in the discussion of the spectral properties of the molecule and also the observed phenomena of the depolarization of the radiation scattered by anisotropic gases.

Measurements of the Amount of Ozone in the Earth's Atmosphere and Its Relation to Other Geophysical Conditions. G. M. B. DOBSON, D. N. HARRISON and J. LAWRENCE. (*Proc. Roy. Soc.*, A768.)—A spectrographic method is available for determining the total amount of ozone in the earth's atmosphere above any point. By grants from the Royal Society five such instruments were made and located in England, Ireland, Scotland, Germany and Switzerland. The Smithsonian Institution also provided for a sixth spectrograph which was sent to Chile.

Two series are given, containing the monthly averages of ozone content above Oxford from February to October, inclusive, for the years 1925 and 1926. In both years the means for September and October are the smallest, though the order is reversed in the second year. Again in both years the average for February to June is distinctly larger than for the four succeeding months, the relation being in the ratio of 6:5. Further in both years the March mean is greater than that of February, while that of April is larger than the March mean. From May onwards there is in general a fall in the value of the mean until September ends. With the single exception of June all the means for 1925 are larger than for 1926. In 1925 the February mean is 142 per cent. of the October mean. In the next year the relation is 120 per cent.

A comparison of the ozone content with sun-spot activity and with diurnal range of the horizontal component of the earth's magnetic field is favorable to the existence of a connection though definite proof is still lacking. "Almost without exception the ozone value is high in marked cyclonic systems and low in anti-cyclonic systems." A marked correlation is shown between ozone content and atmospheric pressures at heights from 9 to 14 km. G. F. S.

The Transverse Magneto-resistance Effect in Single Crystals of Iron. W. L. WEBSTER. (*Proc. Roy. Soc.*, A768.)—Rods were made from single crystals of iron with their axes parallel to the (100), the (110) or the (111) crystal axis. The rods were about 18 mm. long with a square cross-section 1 mm. on a side. A constant current of 1.5 amperes flowed along the length of the rods and the resistance was measured by a differential method. The rods were placed in the air-gap of a large electromagnet capable of giving a field of 27,000 gauss. The lines of force were at right angles to the length of the rods which were rotated about their axes to get different orientations. The presence of a field below 5000 or above 12,000 gauss always causes a decrease in the electrical resistance of the iron that is about in proportion to the field strength. "In the region between, there occurs a rapid change of resistance which is positive when the current is along a cubic axis and negative for the other two directions; and whose magnitude depends on the direction of the magnetic field, vanishing when the field is along a cubic axis." G. F. S.

THE POSSIBILITIES OF DIRECTIONAL RADIO TRANSMISSION.*

BY

DR. J. H. DELLINGER.

Chief of Radio Laboratory, Bureau of Standards, Department of Commerce.

THE conspicuous success of broadcasting frequently gives rise to speculation as to the possibilities in other applications of radio. The waves from a radio station have a natural tendency to spread out equally in all directions. Broadcasting, as we have it to-day, is the culmination of development of the non-directional property of ordinary radio waves. Our subject is therefore sharply set off against broadcasting, and it should be clearly understood that we are entering a discussion of an entirely different field of radio possibilities.

From the early days of radio it has been a goal of scientific imagination and effort to concentrate a beam of radio waves in one direction like a searchlight. The possibilities of individual communication and of navigational aids and controls, if such a thing could be done, are fairly obvious. We are so accustomed to think anything possible with radio that we dislike any thought of limitations. Many efforts have been made to produce radio beams, and a number of very interesting things are being done with such directive properties as radio waves do have.

The directing of radio waves in a very sharply defined beam, like light from a searchlight, is a consummation not likely to be achieved. Nevertheless the approximations that have been and are being made are of very great value. About ten years ago Marconi demonstrated that by the use of a parabolic reflecting arrangement radio waves could be partially directed in a desired direction. With such an arrangement the intensity of the waves along the desired direction was several times that at angles greater than 30° from that direction. With waves of the order of 3 to 10 metres in length a reflecting arrangement of practical size can readily be constructed and the waves can be transmitted distances of several thousand miles. A practical application of this parabolic reflecting system is the rotating radio beacon installed

* Abstract of address given before The Franklin Institute, March 3, 1927.

at Inchkeith, Scotland. This apparatus, on a wave-length of 4 metres, by virtue of a rotating parabolic reflector sends out a radio beam which sweeps around in all directions. By the use of a timing system, mariners having suitable apparatus can determine their direction from this beacon.

From this beginning, reflecting arrangements of improved types have been developed by American and other experimenters, giving the evolution of the so-called beam system. The beam system has been adapted for practical use, particularly by Great Britain. Beam stations are carrying regular radio traffic between England and Canada, and other beam stations are being erected for communication with other parts of the British Empire. In these stations wave-lengths up to as great as 100 metres are used and the reflecting devices are very long, straight rows of antennas. Instead of relying upon a parabolic arrangement of reflectors the currents in the various wires of the straight row have their phases so adjusted as to reinforce radiation in a particular desired direction and diminish it in other directions. These beam stations are of interest and importance because they enable communication to be affected in a desired direction with considerably less power in the transmitting antenna than if ordinary non-directional transmission were used. As previously intimated, they do not confine the waves with extreme sharpness to the desired direction. Their chief advantage is, therefore, an economic one and not one of secret communication.

Certain things have been discovered in the realm of high frequencies or short waves in the last year or two, which give renewed interest to the possibility of using beam methods for communication between one point and another. Very short waves exhibit the phenomenon of skip-distance, that is, beyond a short distance around the transmitting station there may be a zone of several hundred miles where the signal cannot be received and beyond this there will be another zone of a certain width where the signals come in very well. If, now, it were possible to confine such waves sharply along a given line from the transmitting station, the area where they could be received would be limited in two different directions and thus the signals from the transmitting station would be receivable at just one limited area on the earth's surface. By suitable control of the wave-lengths and times of transmission, this system would permit the putting of

a message down at any desired spot on the earth. The trouble is that, as previously stated, beams cannot be sent out anywhere nearly sharply enough to accomplish this with any precision. The dream of radio as a means of individual and secret communication between persons at any two desired places on the earth thus fades away.

It is in the realm of navigational aids that directional radio has attained greatest success. The direction finder is a device now well known to mariners. In its simplest form it is merely a coil of wire which indicates the direction from which a radio wave is coming merely by turning the coil and listening to the variation of the intensity of the received signal. This simple device has many uses. By virtue of its complete lack of response to waves coming from a direction at right angles to the plane of the coil, it can be used to eliminate interference from any particular station. As an aid to ocean navigation it is particularly valuable under conditions of fog. Before the advent of the direction finder, sound signals were the only recourse of the mariner in fog. The distance at which sound signals can be heard is so small that radio is the only actual assistance in fog when great distances are involved. Consequently the radio direction finder is a necessary supplement to the sound and light signals provided in lighthouse services, and lighthouses are being increasingly equipped with radio transmitting systems.

In the rapidly developing realm of air navigation, the direction finder has not been found so practical as in marine use. Fortunately there is another way of utilizing directional radio which bids fair to solve the problem of air navigation in fog. This method, as now under development, takes advantage of the directive properties of a coil antenna. Just as with other types of directive antennas, there is no production of a sharply defined radio beam, but the directive effect is sufficiently great to be usable. At a transmitting station operating as a radio beacon for aircraft navigation, two coil antennas are used, erected at right angles to one another. Each of these sends out radio waves largely directed in the line along which the antenna points. An airplane flying along a line equidistant from these two lines (*i.e.*, the bisector of the angle between the two) receives signals of equal intensity from each. When off to the right or the left of this line, it receives one signal

more intense than the other. A number of ingenious methods have been worked out to take advantage of this variation to indicate to an aviator when he is on the course and when he is off to the left or off to the right of the course which he desires to fly. This system is now being developed as the basis of installation of regular radio beacon service on the civil airways of the United States. The importance of this application of directional radio transmission may be seen from the fact that there are 8234 miles of airways regularly carrying the mail, express, etc., in the United States at the present time.

I have been asked whether the development of directional radio is bringing nearer the possibility of wireless power transmission. There is a vast popular misconception on this subject. It is not the part of a scientist to pronounce anything impossible, but the idea of transmission of substantial amounts of power to considerable distances by radio is ridiculous. It would be possible to build a radio station to transmit enough power to heat a house several miles away, but it would be the most inefficient thing in the world, and not even the wealth of Henry Ford would suffice to pay for the enormous transmitting station that would be required.

Similarly, it is sometimes asked whether radio photography and television are to be materially advanced through directive radio transmission. Here again the answer is essentially negative. Both radio picture transmission and television are thoroughly practical possibilities, but they are adapted essentially to broadcasting rather than to point-to-point transmission by radio. It will unquestionably remain true for a long time to come that the transmission of power or of photographs, vision, etc., from one point to another definitely chosen point can be best carried on by the aid of conducting wires and not by radio. Another application of radio, the remote control of distant objects, machinery, ships, etc., will, on the other hand, be somewhat facilitated through the use of directive radio transmission.

I was interested to read a complaint published by H. G. Wells about ten days ago, over the way in which the world has neglected the possibilities of aviation. He said that the most striking of numerous ill-appreciated gifts available to mankind is flying. There are many reasons for this, and doubtless one of them is the lack of confidence on the part of the average man in the safety of

flying. When the application of directive radio to the airways has been perfected, there is good reason to believe that this condition will be changed. Together with other advances in aviation, radio will help to add sufficient safety to make flying popular.

It may well be that directive radio will never bring wireless power transmission. It has not had conspicuous success in the transmission of individual communications by a beam. But as a service to those who go down to the sea in ships it already has a brilliant record in the saving of many lives; and unquestionably it has a great future as an element in the safety of aviation.

Experimental Confirmation of the Influence of a Low-resistivity Layer Subsoil on the Forward Inclination of Radio Waves. J. E. I. CAIRNS. (*Jour. Washington Acad. Sci.*, May 19, 1927.)—Hack suggested some years ago that the forward inclination gained by radio waves in passing over ground of considerable resistivity would be annulled by the presence of ground water or of a layer of small resistivity situated a fraction of a wave-length below the surface of the earth. A survey of the resistivity of the area surrounding the Watheroo, Australia, Observatory of the Department of Terrestrial Magnetism, Carnegie Institution, made plain that conditions were favorable for putting Hack's conclusion to an experimental test. Two points were selected at which to study the effects upon electric waves. At one the mean resistivity of the surface layer of the earth down to a depth of 5 m. was about 1,500,000 ohms per c.c., while at 60 m. below the surface the resistivity amounted to only 1/1000 as much. At the other spot the resistivity from the surface to a depth of 100 m. nowhere attained a value as large as 400 ohms per c.c. The wave-length employed was 1250 m. Over the two widely differing types of surface the inclination of the waves was found to agree within the limits set by experimental errors and no forward inclination was shown to exist even over the layer of great resistivity. "This is regarded as being more definite proof than has hitherto been given of the effect of ground water or a low-resistivity layer a short distance below the surface, owing to the greater precision of the resistivity measurements of the undisturbed soil."

G. F. S.

The Latent Heat of Vaporization of Benzene at Temperatures above the Boiling-point. J. A. SUTCLIFFE, F. C. LAY and W. L. PRICHARD. (*Proc. Roy. Soc.*, A770.)—The authors felt that a need existed for accurate determinations of the latent heat of the vapor of a liquid so that a test might be made of formulas that have been proposed connecting this quantity with other physical constants of the liquid and its vapor. In brief they supplied a measured quantity of heat, produced by an electric current, to the benzene. From the mass of benzene evaporated and the energy used in evaporating it, the latent heat was calculated. Only about 4 grams were evaporated per hour. At the beginning of the experiment the bomb containing the liquid was at a temperature above its normal boiling-point. The needle valve controlling the efflux of vapor was opened and at the same time the heating current was turned on. The escape of vapor was so regulated that the temperature remained constant, the heat supplied by the current just compensating for the heat spent in vaporizing the escaping liquid.

The boiling-point of benzene is 80.2° C. At 151.8° its latent heat of vaporization is 79.86; at 105.42° it is 89.2. The method seems to bear the promise of similar determination for many other liquids.

G. F. S.

A MECHANICAL SYNTHESIZER AND ANALYZER.

BY

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THE interest of the Riverbank Laboratories in the apparatus and methods described in this paper for the harmonic analysis and synthesis of complex curves, is primarily for their usefulness in the study of sound, but they are of a perfectly general nature and so should be of value in any field in which harmonic analysis is useful.

We have as part of our laboratory equipment, a 40-element analyzer of the Henrici spherical integrator type, made by Coradi, of Zurich, Switzerland. By means of this, there can be obtained the sine and cosine terms of each of five components by tracing over the curve once, and by changes of the wire which serves to drive the pulley wheels and so also by changes in the relative rapidity of rotation of the elements, eight such operations may be performed, thus covering a total of 40 components; that is, the amplitudes are obtained for the sine and cosine terms of all the components having frequencies of from one to forty times that of the fundamental. A complete description of this type of instrument has been given by Miller.[1]

Proof of the correctness of the results obtained by analysis can best be made by the use of a synthesizer which will combine the indicated components of an analyzed curve into a single curve, which can then be compared with the original curve which underwent analysis. Mr. B. E. Eisenhour, of this laboratory, undertook the construction of such a synthesizer, and its compactness, flexibility, ease of adjustment and gear design are such as to warrant description. There will also be discussed the use of this synthesizer for the analysis of curves.

The fundamental requirement of a harmonic synthesizer is that it shall produce a curve which is the algebraic summation of a number of simple sinusoidal curves, the wave-lengths of these component curves being integral subdivisions of the wave-length of the longest, which is also the wave-length of the resultant, or expressed in another way, the frequencies of the harmonic components are to be integral multiples of the frequency of the funda-

mental. Thus the instrument should give y as a function of θ where

$$y = \sum_{k=1}^{n} a_k \sin (k\theta + \varphi_k),$$

the curve to extend continuously from $\theta = 0$ to $\theta = 2\pi$, and k to have all integral values from 1 to n. The amplitude a_k and initial angle φ_k of each component must be capable of separate adjustment.

The above expression is equivalent to expressing y as a function of t, thus

$$y = \sum_{k=1}^{n} a_k \sin (2\pi k f_0 t + \varphi_k),$$

the curve to extend from $t = 0$ to $t = 1/f_0 = T$.

No more satisfactory method of obtaining the algebraic sum of the amplitudes of a number of motions has been found than to combine them in pulls on a cord or chain which passes around pulleys on the moving parts. This principle was used by Kelvin on his tide predicting machine [2] and has been used by many others since. Registration of the resulting summation is accomplished by a pen or pencil on a carriage attached to one end of the chain, thus making a tracing on a paper carried by a table moving uniformly at right angles to the pen motion. The other end of the chain is fixed so the end carrying the pen will have a motion dictated by the motions of all of the moving elements, each producing its effect independently of the motion of the others. A weight keeps an even tension on the chain. Undoubtedly the finest example of an instrument of this type which has been described, is that of Prof. D. C. Miller [3] at the Case School of Applied Science.

The present instrument follows the same general principles. With the arrangement indicated, a sinusoidal curve will be produced by the motion of one pulley if this pulley moves with a simple harmonic motion. The simple harmonic motion of each pulley is produced by the rotation of a circular plate with a projecting pin working in the slot of a sliding element which is constrained by parallel guides to move back and forth along one line. This sliding element carries the pulley and moves with a simple harmonic motion as the plate is rotated with a uniform angular velocity. Each moving element has a different period which is

controlled by the period of the corresponding rotating plate. Helical gears are used to connect these circular plates with a main drive shaft, to rotate the plates at appropriate speeds. The use of helical gears offers a very considerable advantage in economy of space. Also, because of the fact that each tooth slides for some distance along the surface of the tooth which it engages, remaining in substantially uniform contact during this time, and several teeth are thus in contact at any one time, the effect is essentially that of the use of an inclined plane, making the transmission of power smoother and more uniform than is practically possible with the use of spur or bevel gears in which the changes of tooth meshings are necessarily more abrupt than is the case with helical gears.

The synthesizer is about 9½ feet in length, over all. The table carrying the paper is 25 × 36 inches, while the main body of the instrument tapers from a width of 2 feet at one end to 9 inches at the other. The height is a little less than 9 inches except for the pen carriage above the table, where the height is about 11½ inches. The main framework is of cast aluminum, the gears are of steel, while the circular plates are of Sterlite, a hard stainless composition metal which machines well and takes a high polish. Other and smaller parts are of these three metals and brass.

The driving gears are all attached to a main shaft which is rotated by means of a crank at one end, and the driven gears are on the shafts of rotation of the various circular plates. The gear ratio was made 1 : 1 for the highest speed plate, while a reduction in the ratio of 40 : 1 was made for the lowest speed or "fundamental" plate. The gears for the low-speed plates are the largest in size and, for the sake of the appearance of the machine, it was desired to have the space requirements of the gears taper gradually from one end of the instrument to the other. Also it was wished to have as great economy and simplicity in the manufacture of the gears as was consistent with satisfactory performance, and it was accordingly decided to design them so as to require the use of only two cutting hobs in producing the gears, these having diametral pitches of 20 and 28, respectively. The necessary ratios, the gradual taper in sizes and the choice of the hobs dictated the design of the gears. The angles of the teeth vary from 88°–50′ to 45° for the driver gears, and from 1°–10′ to 45° for the driven gears. The diameters vary from about 2 to 5

inches. The gears were lapped in with a light abrasive after their final mounting to eliminate any possible small inaccuracies in the cutting. The axles on which the driven gears were mounted were placed in slightly eccentric bearings so as to allow the closeness of meshing to be perfectly regulated. The main shaft is in four sections which are connected by couplings. This allows the use of the table moving mechanism alone, or of the first twelve elements of the synthesizer, or of the first twenty-eight elements, in case the whole capacity of the instrument is not required. Also the effect of the higher elements may be stopped off at any point by the use of a clamp on the chain.

Each element must be capable of adjustment as to amplitude and phase. The amplitude of motion of the sliding element carrying the pulley is dependent on the distance of the pin from the centre of rotation of the plate, so this distance is made to be adjustable, the adjustment being accomplished in a simple manner by a small knurled nut on the top of the element. The amplitude of the curve drawn will be twice the amplitude of motion of the pulley. A graduated scale on the surface of the plate denotes the curve amplitudes with a vernier for readings to 0.1 mm. The available amplitudes vary from 110 mm. to 40 mm. for the various single elements. There are, however, provided on the instrument two extra rotating elements complete except for any gear connections with the main shaft. These are to make possible the use of ratios of frequency other than the harmonics, or to provide additional amplitude for one or two of the regular series. These two extra elements have available amplitudes of 120 and 115 mm., respectively. The larger of these may be geared to have the rotational speed of any of the elements from 1 to 5, inclusive, and the smaller may be added to any from 1 to 10, inclusive. This makes possible, for example, a total amplitude of 335 mm. for the fundamental if desired, or of 224 for the fundamental together with 212 mm. for the second harmonic. The two spare elements are designed to be driven from the shafts of the first and second elements, respectively, of the regular series.

The phase of the curve at the beginning of the motion of the pen is dependent on the initial angular position of the plate with respect to the maximum displacement position of the sliding element, and this is made adjustable by an arrangement which allows the spindle axis of the circular plate to be rotated with

respect to the gear which is attached to it. A convenient knurled nut at the bottom of this spindle serves to loosen or tighten this attachment to the gear, while a scale of degrees on the bevelled periphery of the circular plate indicates the phase by reference to a line on the main framework of the machine.

The table which carries the paper at right angles to the pen motion is driven from the same main shaft as drives the gears, for the proper relation of the motion of the table to that of the rotating elements is essential. The actual wave-length of the curve produced is determined by the relation of the table motion to the speed of rotation of the circular plates, although the relative wave-lengths of the curve components to each other are determined by the gears alone.

It was desired to have the wave-length of the curve produced by the synthesizer, adjustable over a considerable range, the adjustment not to be limited to a choice between a certain few mixed wave-lengths, but to be continuous between the maximum and minimum limits. To accomplish this, a square bar is pivoted at one end on the frame of the machine, and is made to swing about its pivot by a pin sliding in a slot in the lower surface of the bar, this pin being driven in the direction of the desired table travel by a rack and pinion operated by the main drive shaft. A pin in the bottom of the table works in a slot in the upper surface of the bar and so the table is moved by the movement of the bar. The direction of motion of the table is restricted by side guides to be the same as that of the pin which drives the bar, and so the table motion is proportional to the motion of this pin, the ratio of the motions being determined by the ratio of the distances of the two pins from the point of rotation of the bar. The non-uniformity of the angular motion of the bar does not enter. The distance of the pin in the bottom of the table from the pivot point of the bar can be adjusted by means of a screw thread. Thus any desired relative speed of table travel may be secured and so also any desired wave-length of curve between the limits of 100 and 800 mm.

A friction clutch is provided to release the table mechanism from the main drive shaft so that the table may be easily returned to its starting position. A graduated scale at the crank end of the main drive shaft serves to show the operator the angular position of the shaft at all times. The chain which moves the pen car-

riage passes over a pulley which rotates a graduated scale on which the pen displacement may be read if desired. The chain is a chronometer fusee chain. A picture of the synthesizer is given in Fig. 1. A mirror beneath the instrument allows the gears to be seen.

The instrument makes possible the easy, rapid and accurate summation of a series of forty simple sinusoidal curves of any initial phase and with a considerable range of possible amplitudes. Some twenty possible sources of error in the construction of the instrument have been investigated and found to be of negligible importance. Some examples of curves drawn on this synthesizer are shown in the accompanying figures.

USE OF THE SYNTHESIZER FOR THE ANALYSIS OF CURVES.

In texts on Fourier's series, there are given the integrals

$$\int_0^\pi y \sin kx dx \quad \text{and} \quad \int_0^\pi y \cos kx dx,$$

as the expressions which must be evaluated in order to obtain the coefficients of the sine and cosine terms of the series representing the curve $y = f(x)$. The spherical integrator type of synthesizer mentioned above is designed to obtain the values of these integrals.

The mathematical method of calculating the desired coefficients or amplitudes involves summations instead of integrations. These summations may be performed on a synthesizer with a great saving of time and labor. A discussion of the process will be given, with the limitations of the method.

Let us assume that the curve contains both sine and cosine components, n of each. We may then write

$$y = \sum_{k=1}^{n} A_k \sin kx + B_0 + \sum_{k=1}^{n} B_k \cos kx. \qquad (1)$$

Let us consider the ordinates at the division points found by dividing one complete wave-length of the compound curve into $2m$ equal parts. The spacing between these ordinates would represent π/m in angular terms of the fundamental ($k = 1$). The ordinate at the λth division point will be

$$y_\lambda = \sum_{k=1}^{n} A_k \sin k\lambda \frac{\pi}{m} + B_0 + \sum_{k=1}^{n} B_k \cos k\lambda \frac{\pi}{m}. \qquad (2)$$

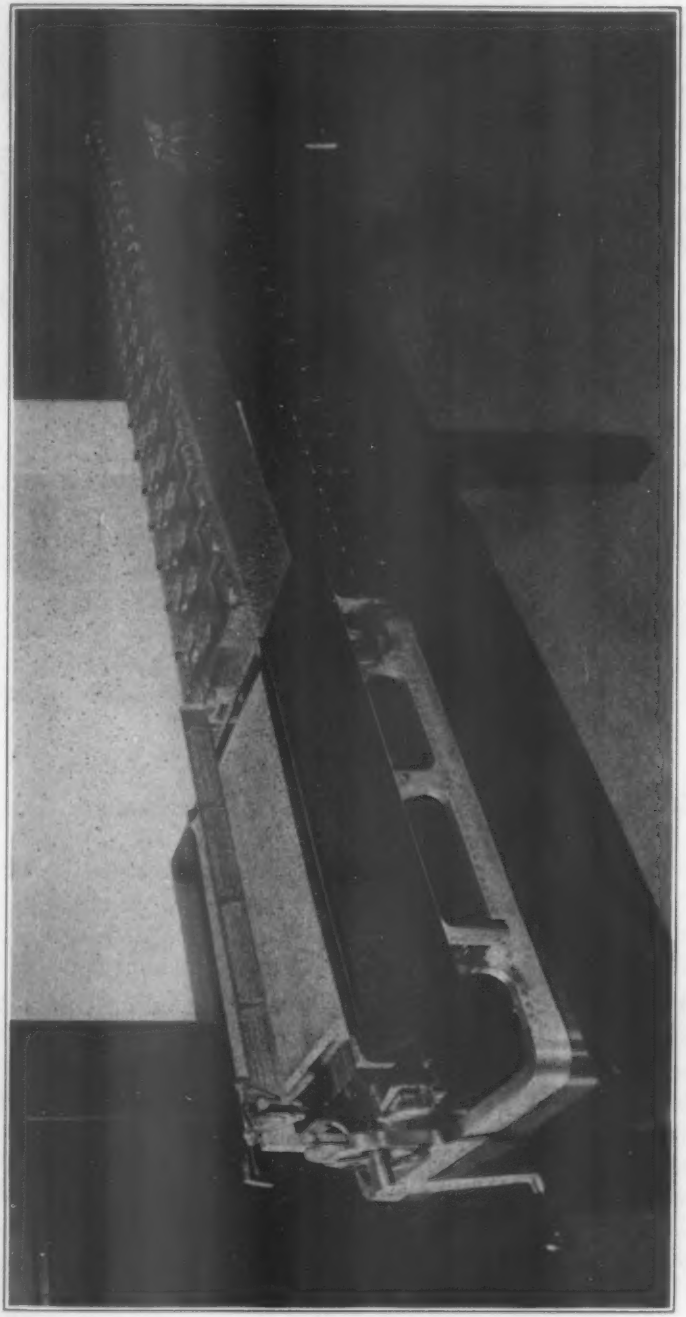


FIG. 1.

Photographic view of synthesizer.

λ has all integral values and there are $2m$ different equations of this type corresponding to values of λ from 1 to $2m$, inclusive. The values of A_k and B_k are to be obtained from the known values of y_λ.

The method of solving for the amplitude of a particular component, say A_r, is to multiply each of the $2m$ equations of the form of (2) by $\sin r\lambda\pi/m$ and add all the equations together, giving

$$\sum_{\lambda=1}^{2m} y_\lambda \sin r\lambda\frac{\pi}{m} = \sum_{\lambda=1}^{2m}\sum_{k=1}^{n} A_k \sin k\lambda\frac{\pi}{m}\cdot\sin r\lambda\frac{\pi}{m} + B_0\sum_{\lambda=1}^{2m}\sin r\lambda\frac{\pi}{m} + \sum_{\lambda=1}^{2m}\sum_{k=1}^{n} B_k\cos k\lambda\frac{\pi}{m}\cdot\sin r\lambda\frac{\pi}{m}. \quad (3)$$

This may be expanded, separating terms having $k=r$ from the other terms for the sake of convenience, resulting in terms of two general types:

$$\sum_{\lambda=1}^{2m}\sin\lambda p\frac{\pi}{m} \quad \text{and} \quad \sum_{\lambda=1}^{2m}\cos\lambda p\frac{\pi}{m}. \quad (4)$$

Byerly [4] gives a formula

$$\sum_{\lambda=1}^{n}\cos\lambda\alpha = -\frac{1}{2} + \frac{1}{2}\frac{\sin(2n+1)\frac{\alpha}{2}}{\sin\frac{\alpha}{2}}$$

and in similar fashion it can be shown * that

$$\sum_{\lambda=1}^{n}\sin\lambda\alpha = \frac{\sin\frac{n\alpha}{2}\cdot\sin(n+1)\frac{\alpha}{2}}{\sin\frac{\alpha}{2}}.$$

This last summation may be shown to be equal to zero if n and α have the values of the sine summation of (4), provided p is an integer. Byerly's formula shows that the cosine summation is equal to zero with the values shown in (4), except that it is indeterminate if $p=2sm$, where s is an integer. However, a consideration of the cosine summation itself shows that if $p=2sm$, the summation will equal $2m$, since each term will be separately equal to 1. So for integral values of p, the sine sum-

* Grover [5] erroneously gives this with a factor of $\frac{1}{2}$ on the right-hand side of the equation.

mation of (4) is equal to zero, and the cosine summation is equal to zero except for the special case noted.

The terms in the expansion of the right-hand side of (3) are then all equal to zero except two types

$$\sum_{\lambda=1}^{2m} A_r \sin^2 r\lambda \frac{\pi}{m} = \frac{1}{2} A_r \left[2m - \sum_{\lambda=1}^{2m} \cos 2r\lambda \frac{\pi}{m} \right] \qquad (5)$$

and

$$\sum_{\lambda=1}^{2m} A_k \sin k\lambda \frac{\pi}{m} \cdot \sin r\lambda \frac{\pi}{m} = \frac{1}{2} A_k \sum_{\lambda=1}^{2m} \left[\cos (k-r)\lambda \frac{\pi}{m} - \cos (k+r)\lambda \frac{\pi}{m} \right], \qquad (6)$$

there being $n-1$ terms of the type of (6), having all values of k from 1 to n, other than $k=r$. (5) is equal to mA_r except for the case of $r=sm$, which will make (5) equal to zero. (6) is zero except for two special cases, first, if $k-r=2\,sm$, then (6) equals mA_k; and second, if $k+r=2\,sm$, then (6) equals $-mA_k$. If $k-r=2\,s_1 m$ and $k+r=2\,s_2 m$, then (6) would be equal to zero.

But if we choose the number of division points, m, in a half wave-length to be not less than n, which is the maximum value of k, or the highest harmonic present in the curve, then (6) will be zero for all values of r. Only the terms of (5) will then be left on the right-hand side of (3), and (3) may be rewritten,

$$A_r = \frac{1}{m} \sum_{\lambda=1}^{2m} y_\lambda \sin r\lambda \frac{\pi}{m}, \qquad (7)$$

which is the well-known formula for mathematical analysis of curves, and which will apply for all values of r except $r=m$.

It is obvious that the value of the amplitude of the sine curve of the mth order could not be obtained from consideration of the amplitudes of the compound curve at $2\,m$ equally spaced division points, for the contribution of $A_m \sin \lambda\pi$ is zero at each of these points, no matter what the value of A_m.

A similar process is followed to get the value of B_r, a particular cosine amplitude. The $2\,m$ equations of the form of (2) are multiplied by $\cos r\lambda\pi/m$ and the equations are added, giving

$$\sum_{\lambda=1}^{2m} y_\lambda \cos r\lambda \frac{\pi}{m} = \sum_{\lambda=1}^{2m} \sum_{k=1}^{n} A_k \sin k\lambda \frac{\pi}{m} \cdot \cos r\lambda \frac{\pi}{m} + B_0 \sum_{\lambda=1}^{2m} \cos r\lambda \frac{\pi}{m} + \sum_{\lambda=1}^{2m} \sum_{k=1}^{n} B_k \cos k\lambda \frac{\pi}{m} \cdot \cos r\lambda \frac{\pi}{m} \cdot \qquad (8)$$

The terms of the right-hand side of (8) which are not necessarily equal to zero are

$$\sum_{\lambda=1}^{2m} B_r \cos^2 r\lambda\frac{\pi}{m} = \frac{1}{2}B_r\left[2m + \sum_{\lambda=1}^{2m}\cos 2r\lambda\frac{\pi}{m}\right] \qquad (9)$$

and

$$\sum_{\lambda=1}^{2m} B_k\cos k\lambda\frac{\pi}{m}\cdot\cos r\lambda\frac{\pi}{m} = \frac{1}{2}B_k\sum_{\lambda=1}^{2m}\left[\cos(k+r)\lambda\frac{\pi}{m}+\cos(k-r)\lambda\frac{\pi}{m}\right] \qquad (10)$$

and

$$B_0\sum_{\lambda=1}^{2m}\cos r\lambda\frac{\pi}{m}\cdot \qquad (11)$$

(9) will be equal to mB_r except for the case of $r = sm$, when it will be equal to $2\,mB_r$. (10) will be equal to zero except for three special cases: If $k + r = 2\,sm$, or $k - r = 2\,sm$, then (10) will be equal to mB_k; if $k + r = 2\,s_1m$ and $k - r = 2\,s_2m$, then (10) will be equal to $2\,mB_k$. (11) will be zero except if $r = 2\,sm$, when it will equal $2\,mB_0$. But as before, if $m \not< n$, the special cases of (10) and (11) will not enter, leaving only (9) to consider.

Omitting the case of $r = m$, (8) may be written as

$$B_r = \frac{1}{m}\sum_{\lambda=1}^{2m} y_\lambda\cos r\lambda\frac{\pi}{m}\cdot \qquad (12)$$

For $r = m$, a value of $2\,B_m$ would be obtained. This is the formula used in mathematical analysis of curves.

The value of B_0 may be obtained by adding together the $2\,m$ equations of the form of (2) and, omitting the terms which will be zero, this will give

$$\sum_{\lambda=1}^{2m} y_\lambda = 2mB_0 + \sum_{\lambda=1}^{2m}\sum_{k=1}^{n} B_k\cos k\lambda\frac{\pi}{m}\cdot \qquad (13)$$

The last term will have a value of $2\,mB_k$ if $k = 2\,sm$, but will be zero for other values of k. If we make the previous restriction that $m \not< n$, we may write

$$B_0 = \frac{1}{2m}\sum_{\lambda=1}^{2m} y_\lambda\cdot \qquad (14)$$

Equations (7), (12) and (13) are then the expressions for obtaining the values of A_r and B_r from the values of y_λ. The

summations indicated are usually performed mathematically by a series of multiplications and additions. The use of the schedules introduced by Runge [6] simplifies the procedure considerably, but the process is laborious at best. The summations may, however, be performed with a minimum of labor on a synthesizer such as described above.

The synthesizer is for the purpose of producing summations of sinusoidal components and so is adapted for summations of the types in equations (7) and (12). For a summation to $2m$ terms, there would be required a synthesizer having $2m$ harmonic elements. Let the amplitude of each be set so that that of the λth element is y_λ. Let all the phase angles be initially set at zero except that those elements requiring negative values of y_λ are to be set at 180°. Let the crank be turned sufficiently to rotate the highest speed or $2m$th element through an angle of $r \cdot 2\pi$. Then the pen will be displaced by an amount

$$\sum_{\lambda=1}^{2m} y_\lambda \sin \frac{\lambda}{2m} 2\pi r \qquad \text{or} \qquad \sum_{\lambda=1}^{2m} y_\lambda \sin r\lambda \frac{\pi}{m},$$

which is the summation desired. If the pen is made to draw a curve, then the ordinate at an angular distance of $r\pi/m$ along this curve will give the required summation for the evaluation of A_r. This summational value must be divided by m as indicated in (7). The complete series of required sine amplitudes may be derived from the curve thus drawn.

Let the rotation of the crank and elements be continued until the initial position has again been reached, that is all the angles again at their original positions. Let the angular settings of all the elements now be advanced by 90°, so that those with positive amplitudes will be set at 90° and those with negative amplitudes at 270°. They will now be in position to draw cosine curves. The pen will be moved by this rearrangement of angular settings by an amount $\sum_{\lambda=1}^{2m} y_\lambda$. From this the value of B_0 may be obtained by dividing by $2m$. This summation may also be performed by simply adding the ordinates, y_λ, algebraically.

With the angular settings as rearranged, a new curve may be drawn and the ordinate at an angular distance of $r\pi/m$ along this curve will be equal to $\sum_{\lambda=1}^{2m} y_\lambda \cos r\lambda \frac{\pi}{m}$, from which the value of B_r

may be obtained by dividing by m, as indicated in (12). As shown above, the value thus obtained for the particular case of B_m' will be twice its true value. The ordinates of this curve must be measured from the position of the pen when all the amplitudes are zero, which is the same as the position at the beginning of the sine process when all the phase angles were zero.

Thus if the ordinates of the original curve at equally spaced division points are set up as amplitudes on the synthesizer, the values of the sine and cosine components contained in the original curve may be obtained from the ordinates of the curves drawn by the synthesizer.

It should be noted that only one assumption has been made in the above process of derivation, which is that $m < n$, or the number of ordinates used per wave-length of the original curve shall be not less than twice the order of the highest harmonic present in the curve. This means, of course, that we must have as many equations of the form of (2) as the number of unknowns we have to determine, the sine and cosine amplitudes of any given harmonic being independent unknowns. Granting that the above condition has been observed, the process contains no approximations, and it should be clearly understood that the use of any additional number of ordinates or any additional number of elements on the synthesizer would add nothing at all to the accuracy of the results. This applies equally well to the usual mathematical analysis.

EFFECT OF THE PRESENCE OF HIGHER HARMONICS.

The result of a violation of the above condition may be calculated. Let there be present a harmonic component of the order q, where $q > m$.

If $q - r = 2sm$, where s is any integer, then (6) is equal to mA_q and (7) would be

$$A_r + A_q = \frac{1}{m} \sum_{\lambda=1}^{2m} y_\lambda \sin r\lambda \frac{\pi}{m} .$$

Also (10) would be equal to mB_q and (12) would become

$$B_r + B_q = \frac{1}{m} \sum_{\lambda=1}^{2m} y_\lambda \cos r\lambda \frac{\pi}{m} . \qquad (15)$$

If $q + r = 2sm$, (8) would be

$$A_r - A_q = \frac{1}{m} \sum_{\lambda=1}^{2m} y_\lambda \sin r\lambda \frac{\pi}{m}$$

and (12) would again transform into (15).

If $q - r = 2\,s_1 m$ and $q + r = 2\,s_2 m$, (7) would be unchanged and (12) would have $2\,B_q$ added to B_r. This condition would require that $q = (s_1 + s_2)m$ and $r = (s_2 - s_1)m$.

If $q = 2\,sm$, then the last term of (13) would be equal to $2\,mB_q$, adding B_q onto B_0 in (14).

So we may write

$$\left.\begin{aligned}
&\frac{1}{m}\sum_{\lambda=1}^{2m} y_\lambda \sin r\lambda\frac{\pi}{m} = A_r - A_{2m-r} + A_{2m+r} - A_{4m-r} + A_{4m+r} - \cdots \\
&\frac{1}{m}\sum_{\lambda=1}^{2m} y_\lambda \cos r\lambda\frac{\pi}{m} = B_r + B_{2m-r} + B_{2m+r} + B_{4m-r} + B_{4m+r} + \cdots \\
&\frac{1}{m}\sum_{\lambda=1}^{2m} y_\lambda \cos m\lambda\frac{\pi}{m} = 2B_m + 2B_{3m} + 2B_{5m} + \cdots \\
&\frac{1}{2m}\sum_{\lambda=1}^{2m} y_\lambda = B_0 + B_{2m} + B_{4m} + \cdots
\end{aligned}\right\} \qquad (16)$$

The left-hand portions of equation (16) are the values obtained by use of the synthesizer for analysis or by the use of mathematical schedules, and the errors entering into the evaluations of A_r, B_r, B_m and B_0 are plainly shown.†

With a synthesizer of 40 elements, used for the purpose of analysis as indicated, we would have $2\,m = 40$, or $n = 20$, and would get values for amplitudes of the sine and cosine components of the harmonic series 1 to 19, inclusive, and also B_0 and B_{20}. If there were present in the curve no harmonic higher than the twentieth, the method should give results with an accuracy limited only by mechanical and observational factors. If there were present in the curve, however, an harmonic of the twenty-seventh order, it would affect the values obtained for the sine and cosine

† Bedell [7] says that if six ordinates are used, and the seventh and ninth harmonics are present, errors will be introduced into the results for the fifth and third components, but offers no explanation.

Since submission of this paper for publication, a reference has come to the author's attention (*I. E. E. Jour.*, **59**, pp. 491–510, 1921) in which A. E. Clayton indicates the influence of the presence of higher harmonics on mathematical analysis, as given above.

amplitudes of the $(2m-27)$th or thirteenth harmonic, the true sine value being decreased and the true cosine value being increased by the respective amplitudes of the twenty-seventh harmonic. The fundamental would be affected only by the presence of the thirty-ninth, forty-first, seventy-ninth, eighty-first, etc., harmonics.

Michelson and Stratton [8] have pointed out how their synthesizer could be used as an analyzer, regarding the summations which were obtained as being approximations to the corresponding integrals, the approximations improving as the number of elements increases. In the present treatment, the summations themselves are shown to give results of perfect accuracy within defined limitations.

A little preliminary calculation will serve to double the number of harmonics which can be obtained from a given synthesizer used for analysis. Since a sine function is an odd function, (7) may be written as

$$A_r = \frac{1}{m} \sum_{\lambda=1}^{m} (y_\lambda - y_{2m-\lambda}) \sin r\lambda \frac{\pi}{m}. \qquad (16)$$

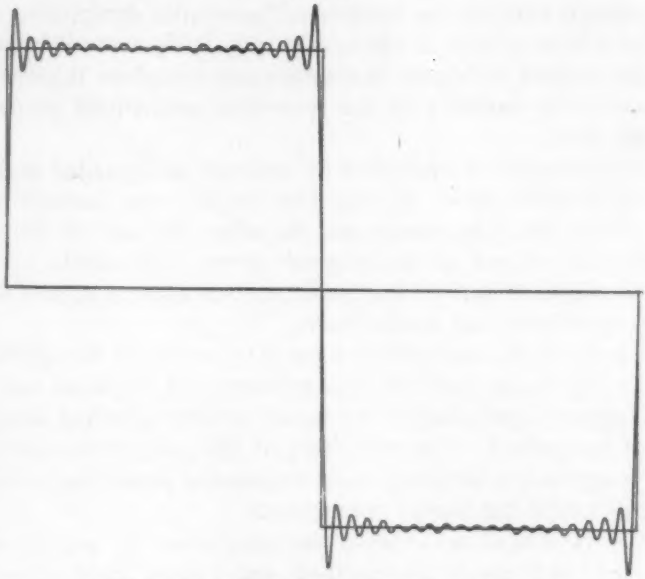
Also (12) may be written as

$$B_r = \frac{1}{m}\left[\sum_{\lambda=1}^{m} (y_\lambda + y_{2m-\lambda}) \cos r\lambda \frac{\pi}{m} + y_0 \right], \qquad (17)$$

with the provision that y_m is to be used at half value in this formula, that is the amplitude of the term corresponding to $\lambda = m$ is to be simply y_m and not $y_m + y_{2m-m} = 2y_m$. The necessity for this provision will appear on comparison of (12) with (17). The term y_0 actually comes into the formula (17) as y_{2m} to which it is equal. It is the ordinate of the initial point of the curve, and a choice of initial point and base line can usually be made to make y_0 equal to zero, if desired. In (16) and (17) there are only m terms in the summations, so with a synthesizer of 40 elements, $2m$ can be made 80, which will allow the determination of the sine and cosine amplitudes of harmonics up to and including the thirty-ninth and also B_{40}. The value of B_0 may be most easily obtained by algebraic addition, following equation (14).

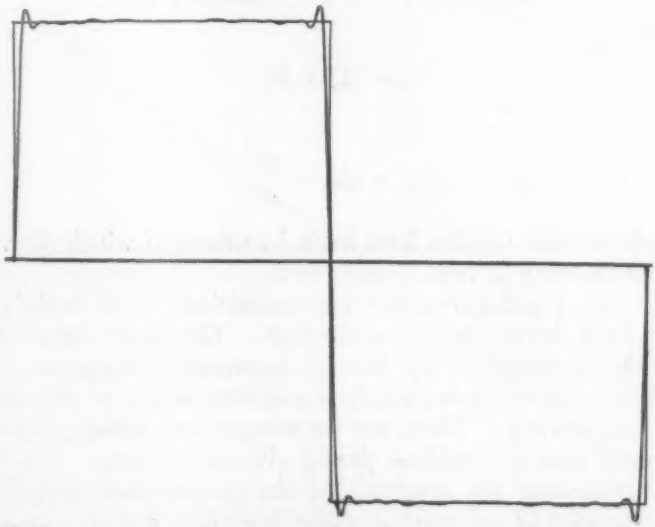
This method of extension of the range of the synthesizer for analysis requires the preliminary addition and subtraction of the ordinates of the original curve in pairs, and also requires that the amplitudes of the synthesizer elements be reset for the determina-

FIG. 2.



Synthesis of first forty harmonic components of the series:

$$y = A \sin x - \frac{A}{3} \sin 3x + \frac{A}{5} \sin 5x - \frac{A}{7} \sin 7x + \ldots.$$



Synthesis of results of a forty-component analysis of the curve represented by the above infinite series.

tion of the values of B_k after the values of A_k have been found. It, however, extends the number of harmonics determined out to a point where, at least in the case of physically recorded phenomena, the neglect of higher harmonics can introduce no real error because of the inability of the recording mechanism properly to register them.

This extension is equivalent to, and may be regarded as, breaking the original curve up into two curves, one comprising the sum of the sine components and the other the sum of the cosine components present in the original curve. Effectively, analyses are then made of half of the combinational sine curve and of half of the combinational cosine curve.

It is found that analysis of a curve by means of the synthesizer is more rapid and demands less concentrated attention and so is less fatiguing than analysis by means of the spherical integrator type of instrument. The stretching of the pulley wire used in the latter machine is a handicap to its mechanical perfection, especially noticeable with the higher components.

If it is wished to express the magnitude of any harmonic component in terms of an amplitude and a phase angle rather than in terms of the amplitudes of the sine and cosine components, this may be done by use of the formula

$$A_r \cos\theta + B_r \cos\theta = a_r \sin(\theta + \varphi_r)$$

where

$$a_r^2 = A_r^2 + B_r^2$$

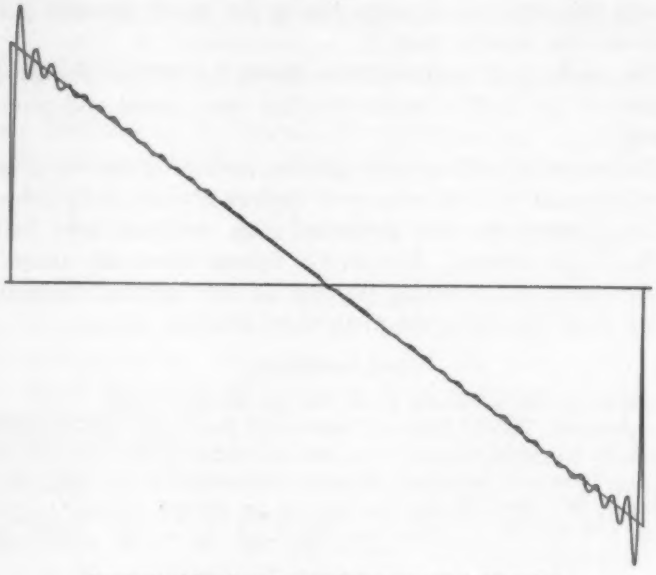
and

$$\varphi_r = \tan^{-1} \frac{B_r}{A_r}.$$

A modified slide rule has been made by means of which the values of a_r and φ_r may be readily computed.

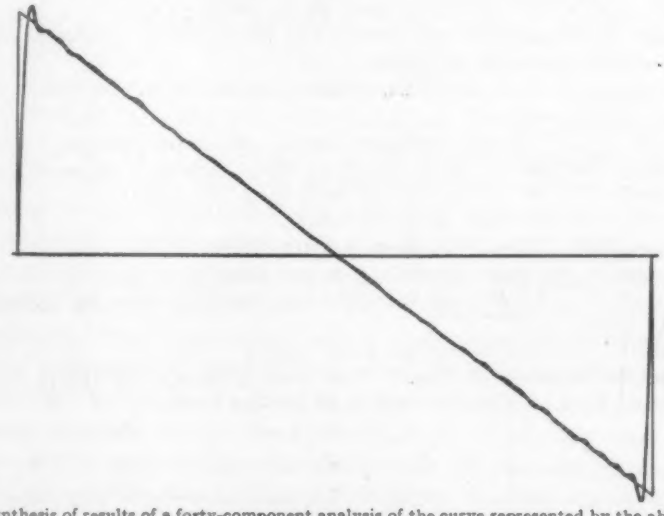
In Fig. 2 are given two approximations to a straight line curve, both drawn on the synthesizer. The upper figure represents the synthesis of the first 40 harmonic components of the theoretical infinite series which if complete would fit the straight line curve exactly. These are the components which a spherical integrator type of analyzer should give on analysis. The lower figure represents the synthesis of the 39 harmonic components as determined by the method above described, and it is seen that

FIG. 3.



Synthesis of first forty harmonic components of the series:

$$y = A \sin x + \frac{A}{2} \sin 2x + \frac{A}{3} \sin 3x + \frac{A}{4} \sin 4x + \ldots.$$



Synthesis of results of a forty-component analysis of the curve represented by the above infinite series.

this is actually a better fit to the curve than is the upper figure, although this cannot be stated as being the case in general. Fig. 3 shows another similar case.

The method of analysis here given, as well as the mode of influence of the higher harmonics, has been tested and proven to be correct.

The appended bibliography gives references to the most important papers and to descriptions of typical devices and methods.

The synthesizer here described was designed and built by Mr. B. E. Eisenhour. Mr. F. G. Tyzzer drew the curves here presented and did a major portion of the work of testing and proving experimentally the method of analysis given.

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NOTES FROM THE U. S. BUREAU OF STANDARDS.*

SPECTROSCOPIC LABORATORY AND EQUIPMENT.

THE facilities of the bureau for spectroscopic research have been enlarged by providing additional space and complete equipment for the study of spectral lines excited in a magnetic field (Zeeman effect). The space consists of three connected basement rooms, the first contains a power plant, the second contains various spectroscopic sources and a large electromagnet, and in the third provision is made for the installation of the bureau's largest spectrographs. The main item of the power plant is a 20-kw. motor generator set to supply the electromagnet with 200 amperes of current at a potential of 100 volts. The electromagnet is suitable for the investigation of Zeeman effects in fields up to 50,000 gausses per square centimetre. This magnet is mounted on a truck so that it may be moved and used in conjunction with any one of the three different spectrographs mounted in the adjoining room, or it may be pushed to one side and any other type of spectrum source (for example, arc in air, vacuum arc, high potential condensed spark, Geissler tube) may be substituted and studied with any of the spectrographs. These last consist of a concave grating, a plane grating, and a quartz prism spectrograph. The concave grating has 640-centimetre radius of curvature and about 70,000 useful lines. It is mounted in parallel light with the aid of an auxiliary mirror and the spectra are photographed on long strips of thin photographic plates clamped to steel rails carefully adjusted to lie in the focal curve. A spectral range extending from 2200 Å. in the first order to 4500 Å. in the fourth order is thus covered by this instrument. The plane grating is provided with collimator and camera lenses of 6 inches aperture and 16 feet focal length; they were made in the bureau's shops from glass which was melted, moulded and annealed in the optical glass laboratory of the bureau. The quartz spectrograph is one of the largest types made; it is used mainly to extend observations in the ultra-violet to about 1900 Å. In addition to the study of Zeeman effects for the purpose of classifying the spectral lines, thus identifying spectral terms and

* Communicated by the Director.

describing the atomic energy states involved in the emission of the lines, these spectrographs will be useful in obtaining more accurate descriptions than those now available for various emission spectra, and their relatively high resolving powers adapt them also to investigations of the complex structure of certain lines.

PHOTOGRAPHIC EMULSION INVESTIGATIONS.

A LABORATORY for the investigation of the photographic emulsion has developed as an outgrowth of the work of the bureau on color sensitivity in its scientific applications, particularly to spectroscopy. Two more papers on color sensitization have been published during the past two years; they deal with sensitization for the entire visible spectrum by one bath, and sensitization for a reversal process by desensitizers. Further studies on the mechanism of sensitization and hypersensitization are in progress. It has been found advisable, as a foundation for practical improvements such as better color sensitivity, to attempt a study of fundamental factors in the preparation of the emulsion. Emulsion making has for various reasons been a highly secret empirical art for over a generation. Reliable data of a quantitative or systematic nature are almost entirely lacking, and the variables are so numerous that much preliminary work has been necessary before the study of any single variable could be undertaken with any confidence that the observed effects could be correlated with the variations deliberately introduced.

The first paper of the intended series [1] has just appeared. Its main interest lies in the correlation of the observed photographic differences with the physical and chemical properties of solutions of the different bromides, such as their solvent power for silver bromide and their tendency to form complex ions with ammonia, both of which affect the precipitation and growth of the sensitive silver bromide grains.

EFFECT OF REPEATED STRESS ON MAGNETIC PROPERTIES.

AN investigation on the effect of repeated stress on the magnetic properties of steel is under way and a paper on the preliminary phases of the subject is in preparation.

[1] "The Photographic Emulsion I Comparison of Emulsions Made with Different Bromides," *Jour. Phys. Chem.*, 31, p. 906, 1927.

It is known that the application of static axial stresses below the elastic limit do not produce a permanent change in magnetic properties after removal of the stress. The results of the present investigation show, however, that repeated bending, such as occurs in the usual rotating beam fatigue test, results in easily measurable changes in magnetic permeability. The changes take place most rapidly in the early part of the test and proceed at a decreasing rate as the test is continued. It has been found that five cycles of stress are sufficient to bring about a measurable magnetic change. The magnitude of the magnetic change depends upon the intensity of the stress and the number of repetitions.

Analysis of the data so far obtained on a single type of steel indicates that the change in magnetic permeability produced by repeated bending is of the same nature as that resulting from the relieval of a compressive stress. This change is probably caused by redistribution of internal stress.

This investigation is being continued, as it now appears very probable that by this method of investigation much can be learned of the fatigue of metals that could be learned in no other way.

THERMAL EXPANSION EQUIPMENT.

THE bureau has received numerous requests for a description of thermal expansion equipment less expensive than that described some months ago. There are many instances where data of highest precision are not demanded and when time and effort can be spared by using an equipment recently assembled at the bureau.

This type has been used in European laboratories for a number of years. Slight modifications have been made by the bureau. A fused quartz tube about 50 centimetres long and 2 centimetres in diameter is closed at one end. The specimen to be tested should be about 20 centimetres long and is placed in the tube. On top of the specimen is placed a fused quartz rod sufficiently long to reach to the open end of the tube. An indicator gauge is fastened to the top of the tube and registers the relative motion between the tube containing the specimen and rod resting on the specimen. The bottoms of the tube and rod are ground concave and the ends of the specimen convex to secure satisfactory contacts. Heating is effected by placing the tube and sample in a

water or oil bath or electrical furnace extending well above the top of the specimen. Low temperatures are secured by using proper cooling baths. A thermocouple placed inside the fused quartz tube near the sample indicates the temperature. The average coefficient of expansion of fused quartz (0.0000005 per degree C.) should be added to the expansivity coefficient secured from this equipment.

A complete description of the apparatus and a comparison of results obtained with this equipment and other types of expansion equipment which the bureau is now testing will be published later.

Photographs showing the general plan of the fused quartz tube expansion device are available for those wishing to install this type of equipment.

Tests indicate an accuracy of better than 2 per cent. in the value of the coefficient for ordinary materials.

REVISED SPECIFICATION FOR PORTLAND CEMENT.

THE United States Government Master Specification for Portland cement has been revised June 10, 1927; copies of the specification now being available at 10 cents each from the Superintendent of Documents. There are several changes in the new test requirements. The 7- and 28-day tensile strengths have been changed from 200 and 300 to 225 and 325 pounds per square inch, respectively, and the specific gravity test has been eliminated. Miscellaneous minor changes have been made in the methods of chemical analysis, as well as in the moulding of the briquets. The pressure exerted during moulding must now be maintained between 15 and 20 pounds. The conditions of storage have been made more definite. The temperature of the mixing water, moist closet, and water in the storage tank are to be kept at 21° C. ± 3°. A few details of the type of testing machine to be used have also been given.

A NEW ACOUSTICAL PLASTER.

A PLASTER which will absorb 10 to 15 per cent. of the sound striking it will remedy the acoustical defects of most of the auditoriums, theatres, and churches having such defects. An investigation is in progress at the bureau with the object of developing such a plaster.

To absorb any appreciable amount of sound energy a plaster must possess a porous surface of such a nature that the sound will penetrate into the plaster and be changed to some other form of energy. The hard, non-porous surface possessed by the ordinary sanded plaster or by the lime-gypsum white finish coat, does not have this porosity, hence practically all sound striking a plaster of this kind is reflected back into the room. This condition gives rise to the objectionable echoes and reverberations which are so often found in public buildings.

There are at present several acoustical plasters on the market which possess the desired property of porosity. With most of them this effect is produced by using a porous aggregate, such as pumice. It is necessary, however, with plasters of this type to use a very closely graded aggregate of fairly large particle size. This results in a somewhat harsh working plaster which is difficult of application and which must be applied with great care and considerable skill in order not to destroy the desired surface.

By adding small amounts of alum or aluminum sulphate and a carbonate ($CaCO_3$, for example) to the calcined gypsum-sand dry mix, it has been found that a plaster may be prepared which possesses the desired characteristic of surface porosity and which may be easily worked and applied. When this plaster is wetted the alum and carbonate react to form carbon dioxide, CO_2, which is entrapped in the body of the plaster in the form of countless minute bubbles of gas. When the plaster is applied to a water absorptive backing, such as an ordinary scratch or brown coat of plaster, the excess water in the wet acoustical plaster is partially removed by the absorptive effect of the backing and the water films surrounding the gas bubbles are broken, leaving a large number of communicating small pores throughout the plaster. These small gas bubbles also serve another purpose in increasing the ease of workability of the plaster and thus making it easier to apply.

Experiments have been made with many different types of aggregate in an effort to find the most satisfactory one for use. Among those investigated may be mentioned sand, pumice, raw and calcined diatomaceous earth, cinders, ground cork, asbestos, tufa stone (a volcanic ash), and several others. Of these pumice, calcined diatomaceous earth, and tufa stone produced the most satisfactory plasters. A plaster containing two parts granulated

tufa, graded to pass a No. 14 and be retained on a No. 40 sieve; one part sand, graded from sieve No. 10 to sieve No. 30; and one part of calcined gypsum, to which was added 2 per cent. by weight of the calcined gypsum of a mixture of calcium carbonate and potassium alum in molecular proportions; and retarder had approximately the sound absorbing properties desired. It is possible to increase the percentage sound absorption by increasing the tufa:calcined gypsum ratio, increasing the tufa:sand ratio, or increasing the particle size of the aggregate. Certain details, such as the method of application, condition of backing, etc., are now being given consideration.

DURABILITY OF STONE.

THE selection of stone for the ordinary type of business structure or residence is usually made principally on the basis of cost and general appearance. In selecting stone for more monumental structures the question of durability should be taken into consideration. It has been learned by experience that some deposits of stone which meet the usual requirements do not possess satisfactory resistance to the destructive action of severe climatic conditions. To construct of an untried stone a monumental building which is expected to endure for centuries but actually begins to disintegrate within a few decades is an expensive experiment.

In this country there are few stone structures which are more than 100 years old. Considering the types of stone so extensively utilized at the present time there are few structures built of them which have stood for a period greater than fifty years. While actual examples of stone structures which have stood for long periods are of the greatest value in estimating the durability of any particular stone, short periods of use have only a relative value in this respect. In order to gain more knowledge of the desirability of present-day building stone, this bureau is conducting experiments to determine the relative resistance to frost action of various types and deposits. Frost action tests, when carried to the point where the specimens are disintegrated, with many deposits of natural stone, require a considerable expenditure of time and hence an important object of this research is to determine, if possible, a more simple means of predicting dura-

bility. A comparison of the frost resistance of many samples of stone with the usual physical determinations such as strength, elasticity, porosity, absorption or relation of absorption and porosity, shows that while such properties may have a bearing on the subject of durability they do not afford a thoroughly reliable criterion for judging this property. A theory has been advanced that if a stone readily absorbs a volume of water greater than nine-tenths of its pore space, it will not be resistant to frost action. This is based on the fact that water increases in volume by one-tenth in solidifying. The present research does not indicate that this theory is reliable. Considerable evidence has been developed that the relation of absorption to permeability is a valuable factor in predicting durability; that is, if the stone offers little resistance to the flow of water through its pores, a high absorption may not be particularly undesirable.

A Technologic Paper will be available at an early date containing the results of frost action tests on the limestones of this country. Work is in progress on the commercial sandstones which will be made the subject of a similar report at a later date.

Destruction of Atoms by Alpha-rays from Polonium. W. BOTHE and H. FRÄNZ. (*Zeit. für Physik*, **43**, No. 7.)—For the elucidation of the mechanism of atomic destruction under α-ray bombardment it is desirable to study the effects of the use of rays of different speeds. For aluminum and nitrogen, Rutherford and Chadwick have done this, employing α-rays of ranges 4.9 to 9.6 cm. Up to the present experiments with slower rays such as those from polonium have been few and have presented somewhat uncertain results. Methodical investigations embracing a large part of the periodic system of the elements have been made only with α-rays from RaC. In this article report is made of the application to atomic disintegration of α-rays from polonium. Their range is 3.9 cm. In respect to rays derived from RaC or ThC the polonium rays have the advantage of being emitted with one speed and, further, of being practically free from admixture with β- and γ-rays. This freedom is of great value in making possible the use of a different method of counting hydrogen particles coming from the atoms disrupted. It has been customary to do this by observing the flashes of light produced on a screen. For this the sensitiveness of the eye must be increased by keeping it in the dark for a time. Long stretches of counting are very wearying. In this present experiment Geiger's point counter is used, whereby the arrival of an H particle is announced by a sharp deflection of an electrometer, which may even be photographed. Thus it becomes possible to let more particles arrive in a given time and to use weaker preparations of the radio-active substance. On the other side of the account is the high susceptibility of the point counter for β- and γ-rays. This renders necessary the use of very pure polonium preparations.

All elements from boron to calcium were subjected to the slow α-particles with the exception of neon and argon. Boron, nitrogen, magnesium and aluminum were certainly disintegrated. In the cases of the other elements, if hydrogen nuclei were produced at all they were far less numerous than with the four elements cited. The range of the H nuclei coming from the four elements was considerably less than the range of those obtained by the use of α-particles of greater ranges. Using particles of range 8.6 cm. Rutherford and Chadwick produced from aluminum H nuclei having a range of 100 cm. The present authors applying their α-particles of 3.8-cm. range got from the same metal H particles with a range of only 16 cm. The results of this series of experiments are not such as to render a decision possible between the divergent data obtained in Cambridge and in Vienna.

The curious observation was made that a piece of copper foil without holes and equivalent in stopping power to 4 cm. air became transparent to α-particles after being heated to 400° to 500° in a vacuum. The change is attributed to an alteration in crystalline structure. G. F. S.

NOTES FROM THE RESEARCH LABORATORY, EASTMAN KODAK COMPANY.*

THE PRODUCTION OF SENSITIZING SPECKS ON SILVER HALIDE GRAINS.[1]

By S. E. Sheppard, A. P. H. Trivelli and E. P. Wightman.

By taking photomicrographs of the same silver halide grains of a photographic emulsion before and after treatment with allylthiourea sensitizer, and again after treatment with alkali, it has been shown that the sensitizer does not react with the silver halide uniformly over its surface but in discrete spots. These spots are distinct in appearance from specks of silver produced by photochemical decomposition of the silver halide. The latter were not formed during the time of making the photomicrographs, at least visible ones were not, when the light beam from the micro-illuminator was filtered with a Wratten E_2 filter. It was also shown that the alkali alone does not produce decomposition specks.

* Communicated by the Director.

[1] Communication No. 282 from the Kodak Research Laboratories and published in *Phot. J.*, 67, 281, 1927.

The Expansion of Charcoal on Sorption of Carbon Dioxide. F. T. MEEHAN. (*Proc. Roy. Soc.*, A770.)—From preliminary experiments it was learned that the volumes of birch and beech charcoals become greater by their absorption of carbon dioxide. The method of investigation was by displacement of mercury, but this was not satisfactory because the liquid penetrated the charcoal and some of it could not be removed. A way of observing changes in length was therefore devised. Cubes of yellow pine three inches in length were heated without access of air for four hours in a muffle furnace at a temperature of about 500°. From the blocks of charcoal thus produced were trimmed cubes two inches in dimension, with the faces parallel to the grain, perpendicular to it and tangential to the annular rings, respectively. A cube was placed in an extensometer in which change of length made a mirror turn. The instrument was placed in a steel cylinder with connections for supplying gas, producing a vacuum and measuring pressure. Deflections of the calibrated mirror were read by telescope and scale. The charcoal was dried out by alternate admission and evacuation of dried air. About two hours was needed after each change of pressure in the carbon dioxide surrounding the charcoal in order to get to the end of changes in size.

The presence of carbon dioxide at one atmosphere caused an increase of length amounting to .2 per cent. of the original length and the coefficient was the same for all three directions. Thus, while wood is æolotropic, the resulting charcoal is isotropic. Similar coefficients were found for a considerable range of pressures. The coefficient is not proportional to the pressure applied but increases most rapidly at low pressures. As the temperature rises the value of the coefficient becomes less. G. F. S.

Gas in Building Construction. (*Gas Progress*, June 15, 1927, **4**, No. 3.)—What is believed to be the first application of manufactured gas to the actual construction of a building is being carried on with pronounced success by the Consolidated Gas Company of New York in the erection of its new office building.

The type of fire hazard that was said to have caused New York's most spectacular blaze in the tower of the Sherry-Netherland Hotel recently, is eliminated by the use of a gas-fired rivet heater. Newspapers claim that the fire was caused by a salamander using solid fuel.

The use of gas eliminates the danger as well as the cost, labor, and inconvenience of passing around live coals.

Other uses for gas in construction jobs are the substitution of manufactured gas in place of acetylene for metal cutting, gas-fired forges for repairing tools, and the melting of tar for waterproofing. Speed is one of the principal advantages of gas in the blacksmith's forge, while the automatic regulation of temperature, made possible by gas, is an important factor in tar melting.

NOTES FROM THE U. S. BUREAU OF MINES.*

WASHABILITY OF FINE SIZES OF COAL ON TABLES, WITH PARTICULAR REFERENCE TO WASHINGTON AND ALASKAN COALS.

By Byron M. Bird.

THE purpose of this work conducted at the Northwest Experiment Station is to make a fundamental study of coal tabling, using float-and-sink tests of zonal products supplemented by screen-sizing tests and chemical analyses. The completed work contemplates a study of the effect of specific gravity of particles and their size and shape on the separation of coal and impurities. The effect of the different variables of the tables such as tonnage, kind of feed, stroke, distribution and quantity of water, cross-slope, longitudinal slope, and so forth, are also being studied in detail. These fundamental data, as they are obtained, are applied to the commercial cleaning of fine sizes of coals. This work, though originally intending to meet the needs of the Pacific Northwest, is gradually expanding into all of the coal fields of the country.

As a result of this investigation to date, methods and apparatus have been developed by which a low-ash coal can be produced efficiently from the bony coals of the Pacific Northwest; also considerable progress has been made in increasing the capacity of the table. The two most important results of the work have been (1) the development of a special system of high, closely spaced riffles for the table, and (2) the development of a hindered-settling classifier possessing the refinements and large capacity necessary for treating coal. Both of these developments have proved remarkably efficacious, either alone or in combination, in increasing both the efficiency and capacity of the table.

Several investigations have been carried on in coöperation with commercial companies; of these the most important was with a large coal company in Colorado. The results of washing a carload of coal at Seattle demonstrated that the ash in the company's washed coal could be appreciably lowered, thus effecting a material saving per year many times the cost of the investi-

* Communicated by the Director.

gation. The results also demonstrated that the improved methods of washing fine coal developed during this work in the Pacific Northwest have a wide application in other fields of the United States.

MANUFACTURE OF FERRIC SULPHATE AND SULPHURIC ACID FROM ROASTER GAS AND FERROUS SULPHATE SOLUTIONS.

By G. L. Oldright and H. E. Keyes.

ONE OF the main problems of the Southwest Experiment Station has been the development of a cheap solvent for leaching semi-oxidized copper ores. A new method of making ferric sulphate-sulphuric acid solutions from waste roaster gases has been discovered, and during the past fiscal year, in coöperation with a mining company, a semi-commercial plant has been successfully operated. Solutions were made that were applicable for the treatment of ore with an acidic gangue containing almost any ratio of the oxide minerals to chalcocite. In addition to the coöperative work, this method of acid making is being used at one plant on a commercial scale, at a second plant on a pilot-plant scale, and at a third on a laboratory scale. Two companies also have made tentative requests for coöperation with the Bureau.

TREATMENT OF COPPER SMELTER FLUE DUST.

By H. E. Keyes and W. A. Sloan.

THE HEAVY circulating load of dust overburdens the settling capacity of many smelting plants, and causes losses, both of which factors could be greatly benefited by improving the present schemes for treating flue dust.

Mineralogical analyses of the copper minerals in the flue dusts of the Southwest smelters were made as well as the usual ultimate analysis of all of the constituents. It was shown that where there were not too high values in precious metals and where the copper in the original ore was oxidized or most chalcocite, that the dust could be leached successfully. The electrical precipitator sometimes builds up enough acid to make the copper in its dust soluble in water alone. Where the copper in the feed was mainly chalcopyrite, it was but little altered in the dust. Some of the rarer elements were also noted to be present in flue dust.

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LIBRARY NOTES.

RECENT ADDITIONS.

Abegg, R., Auerbach, Fr., and Koppel, I.—Handbuch der anorganischen Chemie. Vierter Band, erste Abteilung, erste Hälfte. 1927.

Allen, Alfred H.—Allen's Commercial Organic Analysis. Fifth edition, volume 5. 1927.

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BALY, E. C. C.—Spectroscopy. Third edition, volume 2. 1927.

BEDFORD, CLAYTON W., and WINKELMANN, HERBERT A.—Systematic Survey of Rubber Chemistry. 1923.

CHARBONNIER, P.—Traité de Balistique Extérieure. Two volumes. 1921–1927.

ELLIS, CARLETON.—Synthetic Resins and Their Plastics. 1923.

FERRIER, R.—Quelques Idées sur l'Électrodynamique. 1927.

FOSTER, WILLIAM.—The Romance of Chemistry. 1927.

Franklin Institute.—Report of the Committee on the Explosions of Steam Boilers. 1836.

GARRARD, CHARLES C.—Electric Switch and Controlling Gear. Third edition. 1927.

GEIGER, J.—Mechanische Schwingungen und ihre Messung. 1927.

HEMMING, EMILE.—Plastics and Molded Electrical Insulation. 1923.

LORENTZ, H. A.—Problems of Modern Physics. 1927.

National Research Council of the United States of America.—International Critical Tables of Numerical Data, Physics, Chemistry and Technology. First edition, volume 2. 1927.

POUCHER, WILLIAM A.—Perfumes, Cosmetics and Soaps. Second edition, two volumes. 1926–1927.

SARTON, GEORGE.—Introduction to the History of Science. Volume I, From Homer to Omar Khayyam. 1927.

SAYERS, R. R., and Others.—Experimental Studies on the Effect of Ethyl Gasoline and Its Combustion Products. Report of the U. S. Bureau of Mines to the General Motors Research Corporation and the Ethyl Gasoline Corporation. 1927.

TOLMAN, RICHARD C.—Statistical Mechanics with Applications to Physics and Chemistry. 1927.

World Power Conference.—Transactions, Sectional Meeting, Basle, 1926. Two volumes. 1927.

BOOK REVIEWS.

THE ROMANCE OF CHEMISTRY. By William Foster, Ph.D., Princeton University. xvi–468 pages, numerous illustrations in the text and full-page plates, small 8vo. New York, The Century Company, 1927. Price, $3.

"Romance" in its original meaning is the use of a language other than Latin in the countries in which the latter was the standard tongue. Words frequently undergo material changes in meaning, but it seems very rare that they gain in dignity. They generally fall to a lower grade. Several terms which were in good standing in Shakespeare's day are now not mentionable to ears polite. Others have simply lost some caste and though not absolutely taboo are uncomplimentary and unwelcome. Such a word is "romance." It has acquired a flavor of unreality, indeed of untruthfulness, for if we say of some narrative that it is "romance" we do not place much faith in it.

Doctor Foster does not use the word in this extremely derogatory sense, but in the more respectable one of picturesqueness or attractiveness outside

of the mere practical value. In this mood he has presented an interesting summary of inorganic chemistry. It is claimed to be applicable to plain folk, but while the entire text is clear and informing, the book is really a text-book of the science and will be most useful to the student who is beginning its study. Much information is given about modern applications of chemistry, especially in the industries and the great chemical inventions such as the fixation of nitrogen, hydrogenation of oils, war gases and similar methods of offence.

The book is made attractive by the now popular method of photogravures, all of which are excellent, but the reviewer cannot speak so highly of some of the illustrations in the text which are crude drawings, sometimes more misleading than informing. Such is the illustration on page 24, showing a decomposing cell. It would certainly have been better to use a standard apparatus. The procedure is termed decomposition of water, but should be termed decomposition of sulphuric acid. A good summary is given of the more recently discovered phenomena of radio-activity and elemental degradation. In allusion to the Oppau explosion, it would be inferred by most readers that the exploding material was some well-known high explosive, but it was a mixture of ammonium sulphate and nitrate and the British experts who studied it very carefully were unable to cause it to explode by any initiator that they had at command. The material was made for fertilizer purposes and was stored in very large quantities. It became in time caked so hard that it had to be broken up by the use of low explosives and it had never caused trouble in such case.

The book contains a very large amount of valuable and interesting information expressed in clear, concise language, and will serve as a very satisfactory introduction to the study of chemistry in its essential features.

HENRY LEFFMANN.

ELECTRIC SWITCH AND CONTROLLING GEAR. A handbook on the design, manufacture and use of switchgear and switchboards in central stations, factories and mines. By Charles C. Garrard, Ph.D., M.I.E.E., A.Am.I.E.E. Third edition, revised and enlarged. xiv–783 pages, 23 × 15 cm., cloth. New York, D. Van Nostrand Company, Inc., 1927. Price, $18.

There are certain more or less fixed requirements in the design of an electric power distribution system which, as in all engineering problems, may be met in many different ways, but the evolution of the art has narrowed these methods and the variety of devices which embody them to conform to what past experience has established as good practice. A designer may adopt a very satisfactory method of meeting the requirements of a problem, yet an intrinsically better solution may be available, and an efficient means of surveying the field is an invaluable aid to the designer.

In the particular field of electric switch and controlling gear, the present work aims to provide this sort of aid in stating and illustrating the principles underlying their design and construction. The descriptions are rather illustrative as representative examples or as especially demonstrating some general principle than of discussing specified apparatus. The author has proceeded according to the plan of making the treatment as simple as is consistent with accuracy. The work is generously illustrated by 649 line-cuts and half-tones

of very satisfactory quality, especially the line-cuts, several of which are on inset plates. The mathematical features, the author states, have been kept as simple as possible and only those which have been thoroughly tested in practice have been employed.

The main divisions of the work are as follows: List of symbols used. Chapter I, Materials and manufacturing methods. Chapter II, Apparatus for making and breaking electric circuits. Chapter III, Apparatus for preventing a dangerous condition of the current flowing. Chapter IV, Apparatus for regulating the amount of current. Chapter V, Apparatus for starting and controlling electrical running machinery. Chapter VI, Low and medium tension switchboards. Chapter VII, Hight-tension and extra high-tension switchboards. Chapter VIII, Apparatus for protecting electrical machinery against abnormal electrical conditions. Appendices on specifications, codes and regulations.

The book, being of British origin, naturally cites apparatus manufactured in that country, but frequent references are made to devices made in this country, and since the work is primarily for the purpose of setting forth methods and principles, British practice which predominates should not diminish its value to American designers. Even for a large and well-executed work, neither unusually difficult type-setting nor features of the cuts beyond their considerable number account for the high price of the book.

To designers of electrical switchgear who must devise methods and choose apparatus, the work should prove a most valuable source of information.

LUCIEN E. PICOLET.

THE FUNDAMENTALS OF ASTRONOMY. By S. A. Mitchell, Ph.D., LL.D., and C. G. Abbot, M.S., D.Sc. xi–307 pages, illustrations, 8vo. New York, D. Van Nostrand Company, Inc., 1927. Price, $3.

Edward Young, known almost entirely by the pessimistic sentiments of his "Night Thoughts," says that "By night the atheist half believes a God" and further "An undevout astronomer is mad." The book in hand takes in the preface nearly the same view, holding that astronomy has high cultural value and lifts the mind above superstition and bigotry. This view may be challenged but it is not wise to enter into a long discussion thereof. History tells us and present-day experience exhibits a very considerable amount of superstition in connection with astronomical phenomena. Astrology was a very early practice of civilized man and still continues to have votaries. Even in very recent days, reputable periodicals have thought it not beneath their dignity to publish horoscopes. Cumont, in his work on "Astrology and Religion," says that the ancient Greek philosophers were not believers in any influence of the stars over human beings. They regarded them simply as balls of fire. Oriental doctrines, however, crept in and spread over the Greek and Roman world and have continued until the present time to influence the minds of many to the effect that stars and their positions have some relation to human futures.

There is a considerable number of works on astronomy now available and journals and newspapers furnish the public with a large amount of information. The present book is somewhat different from the usual run of these

manuals. It deals more extensively with accessory phenomena, such as the method of utilizing the sun's heat, the apparatus now employed in the astronomical observatories and the physical phenomena of planets, fixed stars and other celestial bodies. It is very liberally illustrated and very well printed. The chapter on time, for instance, is a valuable explanation of the different types of time and in connection with this an account is given of the principal calendars of former days and of the present. It is stated that England adopted the Gregorian calendar in 1750, but this was the date of the passage of the act providing for the change; the actual change did not occur until 1752, when the day after the second of September was termed the fourteenth, thus cutting out the eleven days of error in the old calendar.

Looking over a modern work on astronomy, one turns of course to Mars. The authors of this volume are slightly influenced by the reports from the Flagstaff Observatory, dealing somewhat leniently with the theories of intelligent life on that planet. Curiously they speak of a race of "human beings" on the planet. This seems a very unwise expression, but we are told that Lowell went much further, claiming that the Martians were more advanced in civilization than the inhabitants of this earth. Statements of this kind have no place in any scientific publication. The question as to what is civilization would have to be decided and then what constitutes a higher stage. It is pleasant to notice that Barnard doubted the existence of the so-called canals on Mars. The chapter taken altogether is commendable for its very cautious presentation of the Lowellian views. It should, however, have been mentioned that the word "canal" is a mistranslation of an Italian word applied by Schiaperelli to the markings that he saw. He called them "canali," which means channels, but by substituting the word "canals," Lowell and his followers have made a better case than otherwise would exist.

The book can be recommended to all who wish to obtain a knowledge of modern astronomy, of some of the methods of research therein and of some of the applications of astronomical phenomena to human uses.

HENRY LEFFMANN.

EMINENT CHEMISTS OF OUR TIME. By Benjamin Harrow, Ph.D. Second edition, enlarged. xx–471 pages, illustrated, 8vo. New York, D. Van Nostrand Company, Inc., 1927. Price, $3.

With this enlarged second edition, Doctor Harrow has fulfilled his original intention. The first edition consisted of an account of the "lives" of some eleven chemists of the modern era although not necessarily living, for even since this edition has come from the press, one of those whose life is portrayed, Remsen, has passed on. In addition to the biographical sketches, we find in this new edition a detailed account of the researches and labors of the chemists. A new feature is a copious list of references to the original literature, which immeasurably increases the value of the book not only for general reading, but also for didactic purposes. Indeed we actually have two splendid volumes bound as one, and it is to the credit of both the author and the publisher for presenting such a volume to the chemical public.

GEORGE WESLEY PERKINS.

KINEMATICS OF GEROTORS. By Myron F. Hill, A.B., A.M., Member of Society of Naval Architects and Marine Engineers, N. Y. Academy of Sciences, Harvard Engineering Society. Second edition, 44 pages, 23 × 15 cm., cloth, illustrated. Philadelphia, The Peter Reilly Company, 1927.

The substitution of rotating parts to replace the piston and cylinder of fluid-pressure apparatus has proved a prolific field for the exercise of the ingenuity of inventors in every quarter where motive-power is employed. Among those which have survived to a restricted extent the test of practical use is that type known as "gear pumps," or blowers operating on a similar principle. That gear pumps, consisting simply of two gears mounted within a closed casing which is provided with inlet and outlet orifices, are of considerable antiquity is attested by a statement in the old treatise of Ewbank that a model of such a pump was found in the private museum of a French collector who was born in 1593. For light service such as pumping oil for lubricating the cutting points in machine tools and for the cooling system of automobiles, these pumps have had considerable vogue, though in the latter service they were soon displaced by centrifugal pumps. More ambitious designs like the Greindl pump and the Root and Connersville blowers have had a longer lease on life.

Under the title "Gerotors," which presumably is a coined word, the author of this monograph describes a system of rotors for a gear pump which consists essentially of an internal gear and pinion so proportioned that their addendum circles are tangent at a point diametrically opposite the point of contact of their pitch-circles with the result that with suitably shaped teeth each tooth of the pinion maintains at least one point of contact with an adjacent tooth of the internal gear. Accuracy of form to secure this result, the author further states, is obtained by first generating the annular tooth shape by the conjugate process (that employed in the now extensively used gear shaper) with a circular milling-cutter as master tool, and then utilizing the annular cutter so formed for generating the pinion in the gear shaper. Teeth so formed being conjugate operate correctly but apparently they could be made epi- and hypo-cycloidal curves and, bearing in mind the double generation of such cycloidal curves, definite points of contact all around could be geometrically assured.

Only the rotors and their diagrams are shown. There are certain other mechanical difficulties incident to this design of gear pumps than the action of the rotors, notably the pressure between the faces of the rotors and the end-faces of the casing. In a high-pressure pump of practical size this is likely to prove decidedly troublesome and to require exceedingly heavy construction to provide against distortion and insure fluid tightness. Trouble of that kind developed with another annular gear pump which was under development a dozen years ago. It was of considerable size as such pumps go and consisted essentially of a truly elliptical gear rotatable about its centre, which drove a set of planetary pinions. These pinions were maintained in proper relation to the elliptical driver by means of a surrounding fixed annular gear of non-circular form. It was an ingenious device and skilfully nursed into operative condition but like many others, it seems to have taken the journey. The

successful test of a pump with rotors three-eighths of an inch thick and three and three-eighths of an inch in diameter (presumably the outside diameter of the annular rotor) is described. That is a very small affair and too small from which to draw general conclusions on the performance of such pumps.

LUCIEN E. PICOLET.

LES LAMPES A PLUSIEURS ÉLECTRODES ET LEURS APPLICATIONS EN RADIOTECHNIQUE. Par J. Groszkowski, traduit et adapté du Polonais par G. Teyssier, Ingénieur Radio E. S. E. xvi–348 pages, 23 × 16.5 cm., paper, illustrated. Paris, Étienne Chiron, 1927. Price, 40 francs.

Twenty-three years have elapsed since Fleming and Wehnelt defined the phenomenon of the emission of negative charges from an incandescent filament, quite a long period in this age of rapid evolution in physical science. Development and refinement still continue, but the theory of the functioning of the tube is now well enough established and only slight modifications in its theoretical aspects need be expected to meet minor changes of arrangement in the course of progress of the art. The author has accordingly limited the present treatise to fundamental layouts.

The work is based upon lectures on the vacuum tube at the electrotechnical department of the École Polytechnique of Warsaw. It covers the study of the physics of cathode tubes (particularly three-electrode tubes) and their application as detectors, amplifiers and generators. The illustrations are clear and numerous and the forms of the tubes which are analyzed are representative of the practice of many different countries. The chapter headings are: Electron emission by incandescent bodies; Two-electrode tubes; Three-electrode tubes; Three-electrode tube as a detector; Three-electrode tube as an amplifier; Three-electrode tube as a generator; Various arrangements and different types of tubes with several electrodes. A bibliography of considerable length is appended. The work should prove of value both for reference and systematic study to readers who are proficient in dealing with the type of mathematics employed in works on electrical engineering.

LUCIEN E. PICOLET.

A SHORT HISTORY OF PHYSICS. By H. Buckley, M.Sc., F. Inst. P. xi–263 pages, small 8vo. London, Methuen and Company, 1927. Price, 7s. 6d.

Within the compass of this convenient volume a large amount of information on the history of the nearest relative to chemistry is to be found. The story of chemistry itself has been told by a considerable number of writers, beginning with Kopp's classic, but concise statements of the development of physics are not so numerous. Physics is, indeed, steadily drawing chemistry into its own field, and before long there may be a break in the latter science by which its practical phases will be quite notably separated from its theoretical principles. It is noteworthy that many of the most recent epoch-making advances in chemistry have been made by physicists, and present-day text-books of chemistry give large space to physico-mathematical data that are of only secondary interest to those engaged in analytic work in connection with industrial, biologic or clinical applications.

The book goes back, of course, to the Greeks, the source of almost all our science, literature and art. Their "marble nymphs" are still the most attractive features of our museums; their "scrolls of worldly lore" are the germs of our prose and poetry, and though much of their science has been relegated to the limbo of rejected things, we are deeply impressed with what they accomplished with limited means. In the physical sciences they gave us the word "atom"; in significance it is true now much changed, and the word itself etymologically erroneous, but the atom is still the unit of chemical action, and in analytic and works laboratories chemists can forget all about isotopes and isobars, and ignore the theories as to the internal arrangements of protons and electrons.

The early Greek investigators are termed "philosophers," lovers of wisdom, and in a much later period, up to the time of many now living, the department of physics was called "natural philosophy." Taine in one of his essays says somewhat satirically that "In England a barometer is still called a philosophical instrument."

The author points out the reasons for the failure of the Greek and Roman philosophers to forward real scientific investigation. It was due in large part to the lack of interest in the applications of the knowledge so acquired. The coöperative relation between pure and applied science that is so characteristic of to-day was unknown; even a suggestion of it was condemned in ancient times. Schiller has a short poem illustrating this fact. A youth applies to Archimedes for instruction and instances some of the master's practical accomplishments as having led to the desire. The master tells him that such results are secondary, science must be studied for itself alone, not for results, and closes by saying:

"Wer um die Göttin freit, suche in ihr nicht das Weib."
(Who woos the goddess must not hope the wife.)

The later developments of physics, relativity, the quantum theory and the views as to the structure of the atoms are presented as closing chapters. The book is not merely a chronicle of events, but a dissertation on many of the important questions of physics. The intimate connection of the science with chemistry is duly considered and the names of Boyle, Black, Priestley, Lavoisier and of many other chemists are mentioned. HENRY LEFFMANN.

GENERAL CHEMISTRY, THEORETICAL AND DESCRIPTIVE. By Thomas P. McCutcheon, Ph.D., University of Pennsylvania, and H. Seltz, Ph.D., Carnegie Institute of Technology. x–415 pages, small 8vo. New York, D. Van Nostrand Company, 1927. Price, $3.50.

It is difficult to present anything new in a manual of general chemistry. The field has been extensively exploited, and many types of books are available, from pocket volumes to bulky tomes and from popular treatises to texts adapted only to those trained in the accessory sciences of physics and mathematics. The present work is limited to inorganic chemistry, and is mainly intended as a companion to a course of lectures and quizzes with laboratory exercises. It lays down the general principles of the science as now taught

and gives a large amount of descriptive matter. The treatment of principles covers nearly half the book. Although the periodic system is illustrated and explained, the descriptive sections take up the elements largely in the order of their practical importance, oxygen and hydrogen being first and then their compounds, after which the halogen and sulphur groups are presented. Though a considerable amount of theory is presented in the first third of the book, some important points, such as valence, equations and nomenclature are included in the descriptive part. It is much more satisfactory to discuss these points in the initial period of instruction. The terminology and basic principles of a science should be given to the student in their main features before taking up the practical side.

So far as the general nature of the book is concerned, it is a satisfactory summary of inorganic chemistry. The newer views are set forth in much detail and very clearly. The authors lean to the Rutherford-Bohr view of atomic structure and some of the simpler atoms are illustrated by diagrams. Much attention is given to those phases of physical chemistry which are closely related to chemistry proper.

Mendelèeff is given credit of being the first to observe the periodic relations of the elements. This is a mistake. Döbereiner, in 1816, noted relations between atomic weights and properties and in 1860, a Philadelphian, Matthew Carey Lea, published several papers in the *American Journal of Science* on such relations. In 1865, several years before the Russian chemist announced his system, Newlands had presented to the Chemical Society his "law of octaves." His communication, as is well known, was refused admission to the publications of the Society, but has since been properly appreciated. It is as unfair to credit Mendelèeff with the whole honor of discovery of the periodic system as it is to credit the Wright brothers with the whole honor of inventing the airplane, and ignore Langley's patient and self-sacrificing work.

HENRY LEFFMANN.

X-RAYS, PAST AND PRESENT. By V. E. Pullin, Director of Radiological Research, Research Department, Woolwich, and W. J. Wiltshire, Research Department, Woolwich. 229 pages, 21.5 × 13.5 cm., cloth, illustrated. New York, Van Nostrand Company, 1927. Price, $4.50.

There are few scientific achievements of modern times that have appealed more forcibly alike to the professional and layman than the discovery of X-rays. Incredulity occasioned by their mysterious power of penetrating objects opaque to visible light was quickly dispelled by the relative ease with which they could be produced with ordinary physical laboratory facilities. The account of the discovery closely parallels that of Daguerre's discovery of the developing power of mercurial vapor upon the latent image of the silver iodide plate. For many years prior to Röntgen's discovery, X-rays had been produced in the course of notable investigations on the discharge of electricity through gases, some of whose effects were only later recognized as the result of X-rays, but Röntgen was the first to trace that cause from an observed fluorescent effect.

The authors have apparently omitted nothing in setting forth from their

earliest beginnings the researches with their numerous and diversified side-lights which culminated in a discovery of rare value to medical science and of far-reaching importance in the very fundamentals of physical science. That achieved, they proceed with equal thoroughness to trace in detail the subsequent development of the diverse applications of these rays under the general divisions: "X-rays and the atom," "X-rays and crystal structure," "X-ray apparatus," "X-rays in medicine and surgery" and "X-rays in industry."

It is all told with a facile pen worthy of a professional scribe and with a precision of statement and clarity of discussion which bears the hallmark of scientific worth. The book is addressed to the non-professional reader, but it is safe to say, once it is opened by a professional reader, he will read on.

LUCIEN E. PICOLET.

APPLIED X-RAYS. By George L. Clark, Ph.D., Assistant Professor of Applied Chemical Research, Massachusetts Institute of Technology. xiii–255 pages, 23 × 15 cm., cloth, illustrated. New York, McGraw-Hill Book Company, Inc., 1927. Price, $4.

On the same subject as the previously noticed work, Doctor Clark's book is primarily a presentation of the present state of the X-ray art, or as he expresses it, "X-rays as a new tool for industry." Ample consideration is given the underlying principles from which present-day developments and applications have been derived. The spectroscopy of X-rays which has been of profound value to the physicist towards establishing a theory of atomic structure and of the interrelationships between matter and radiant energy are wisely dwelt upon at length. The striking work of the Braggs and von Laue on the application of X-rays to the analysis of crystal structure is also given generous space.

The subject-matter falls into a natural arrangement in three parts. The first eight chapters present the fundamental physics of X-rays; chapters ix to xii cover the properties and the applications of the radiation as such; the remaining chapters, xiii to xxii, deal with the analysis of the ultimate structures of materials.

In addition to its deductive and analytic character the work contains many bibliographical references and a large collection of quantitative data. It is quite the sort of book for the student who takes the subject seriously. Mention is due to the fine typography and excellent cuts which the coated paper upon which the work is printed has made possible. LUCIEN E. PICOLET.

TITANIUM, with special reference to the analysis of titaniferous substances. By William M. Thornton, Jr., Johns Hopkins University. A. C. S. Monograph Series, 262 pages, illustrated, 8vo. New York, The Chemical Catalog Company, 1927. Price, $5.

This book, as with so many of its series, illustrates the advance of specialism in chemistry. Fifty years ago a few lines would have been given to titanium in the ordinary text-books, and only the most comprehensive analytic manuals would have contained methods for its determination. In fact, for a long time routine analyses of rock, especially silicates, were made without

noting the titanium; it was included with the separated silica. As with many other elements, accurate analytic methods and zealous search, spurred by economic interests, have revealed that it is not to be considered rare in the strict sense of the word. The same has happened with cerium and some of its cognates and seems to be beginning to happen with radium, for the discoveries in the Congo have added materially to the world's store of that remarkable and valuable element.

There is, however, another and a more serious thought suggested by the book. In the days when titanium was recognized as a member of the group of elements, the name was given without special significance. In the same way, cerium, yttrium, beryllium, rubidium, cesium, thallium, indium, barium, iodine and many others were named; merely fantastic allusions to mythologic deities or to some noticeable property, weight, color, or spectroscopic character. Locality, if considered at all, was merely the place of occurrence or the particular mineral deposit. A change has come over scientists. The growing spirit of nationalism, evident years ago in moderate form, but since the war much more intense, has led to names which celebrate the nation of the finder, or some specific and important locality thereof. Hence our list has been increased by hafnium, gallium, lutecium, rhenium, masurium, scandium, and, last but not least, Trebonius, illinium. Deeply significant are the names rhenium and masurium, referred to districts respectively on the east and west of Germany before the war and now somewhat deleted by the treaty of 1919. Is this a notice to the world that Germans consider that these districts are only out of their control until the hour of revenge has struck? It seems so.

> "What though the field be lost?
> All is not lost; th' unconquerable will
> And study of revenge, immortal hate
> And courage never to submit or yield."

"Revenons à nos moutons" which takes us from germanium to gallium, but the goal is titanium, and the merits of this book must be discussed. It is an excellent work. Though the principal purpose is to set forth the analytic methods, much valuable information is given concerning the industrial applications of the metal, its more important sources, its atomic weight and typical compounds. Its high affinity for nitrogen is a peculiar property. Many years ago titanium nitride and cyanide were found in considerable amount in slags from iron furnaces in the Hartz mountains, and were for a time considered as the metal, but Wöhler showed the true composition. The industrial application of the compounds of titanium are numerous. It enters into pigments, alloys, dyes and accessory materials, refractories and enamels, inks, catalysts and many other incidental uses, all of which are fully and clearly set forth in this book. A novel and interesting application is that of titanium tetrachloride for producing smoke screens. Smoke screens from various sources have already been much used in war, and have of late become quite familiar in "sky writing." The value of the titanium compound is discussed in comparison with the corresponding compounds of tin and silicon. Tin not being available in appreciable amount in the United States, the titanium compound would seem to have the choice but silicon tetrachloride, the author states, is

more satisfactory in some respects. The analytic section is extensive and comprehensive and may be depended upon as covering all practical methods at present known. The work is for the specialist, of course, but will be a very useful addition to his laboratory. Incidentally the reviewer wishes to commend the method of the publishing firm in omitting the unnecessary paging of the preliminary portion with Roman numerals. The book begins with page 1 as it should. HENRY LEFFMANN.

ADVANCED PRACTICAL PHYSICS FOR STUDENTS. By B. L. Worsnop, B.Sc., and H. T. Flint, M.Sc., Ph.D., Lecturers in Physics, King's College, London, Examiners in Physics, University of London. vii–640 pages, illustrations, 8vo, cloth. New York, E. P. Dutton and Company. Price, $8.

Highly practical book. Irrespective of the students for whom it is supposedly intended, this well-written text-book would form a valuable addition to the personal library of any research engineer. The exceedingly attractive manner in which it is published is rather on the par with that in which the fundamental conception of the authors was carried out by them.

N. W. AKIMOFF.

AN INVESTIGATION OF CERTAIN METHODS FOR TESTING HEAT INSULATORS. By E. F. Grundhofer. The Pennsylvania State College Engineering Experiment Station Bulletin No. 33. 71 pages, illustrations, 8vo, paper. State College, Penna., 1925.

An exceedingly useful monograph. No mechanical engineer dealing with heat in any of its applications should be without a copy of this pamphlet. The very complete bibliography on the question of heat transmission makes it quite unique. N. W. AKIMOFF.

DER GENAUIGKEITSGRAD VON FLÜGELMESSUNGEN BEI WASSERKRAFTANLAGEN. Von Prof. Dr.-Ing. A. Staus. 35 pages, illustrations, 8vo, paper. Berlin, Julius Springer, 1926.

This interesting pamphlet covers the theory and the construction of some of the more advanced types of current meters more in detail than average text-books of hydraulics do. The extremely neat manner in which it is published facilitates the study of the highly practical subject, in which the accuracy of the measurements appears to be indeed the starting-point.

N. W. AKIMOFF.

PROCEEDINGS OF THE FIRST INTERNATIONAL CONGRESS FOR APPLIED MECHANICS, DELFT, 1924. Edited by Prof. Ir. C. B. Biezeno and Prof. Dr. J. M. Burgers. xxii–460 pages, illustrations, quarto, boards. Delft, Technische Boekhandel en Drukkerij J. Waltman, Jr., 1925. Price, $7.

From the advance copy, containing in abridged form most of the papers presented, and published in full in this most remarkable volume, those interested in applied mechanics were well able to form an opinion as to the general character of the congress and of its work. It would be quite out of question

to review the fifty papers, excellently edited and published in the form of this large volume. As a general remark, however, the fact can be mentioned that all of these valuable papers cover subjects of most immediate interest in connection with rational mechanics, elasticity and, especially so, hydro- and aero-dynamics. Announcement of the preparation of the volume of the Proceedings of the Second Congress was recently made.

N. W. AKIMOFF.

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS. Report No. 261, Resistance and Cooling Power of Various Radiators. By R. H. Smith. 16 pages, illustrations, quarto. Washington, Government Printing Office, 1927. Price, ten cents.

This report combines the wind-tunnel results of radiator tests made at the Navy Aerodynamic Laboratory in Washington during the summers of 1921, 1925, and 1926, and submitted for publication to the National Advisory Committee for Aeronautics, November 29, 1926. In all, thirteen radiators of various types and capacities were given complete tests for figure of merit. Twelve of these were tested for resistance to water flow and a fourteenth radiator was tested for air resistance alone, its heat-dissipating capacity being known. All the tests were conducted in the 8 by 8-foot tunnel, or in its 4 by 8-foot restriction, by the writer and under conditions as nearly the same as possible. That is to say, as far as possible, the general arrangement and condition of the apparatus, the observation intervals, the ratio of water flow per unit of cooling surface, the differential temperatures, and the air-speeds were the same for all. Also, for reasons of comparison, the L/D value of 6, which was assumed in the 1921 tests as the L/D of the airplane using the radiator, was also used in the more recent tests.

No attempt is made to enter upon the theory of heat dissipation. Only the actual test results are given and reduced to coefficient form. The precision of the tests as representative of full-flight performance is definitely known only in the case of the HN-2. The McCook Field full-flight performance and the Navy tunnel performance of this radiator agree within about 3 per cent.

Since this full-flight test was made with unusual care and since the wind-tunnel tests on all the radiators were made not only accurately but also at almost full scale, it would seem probable that these tests represent quite accurately the full-flight performances in actual service.

PUBLICATIONS RECEIVED.

X-rays, Past and Present, by V. E. Pullin and W. J. Wiltshire. 229 pages, illustrations, plates, 8vo. New York, Van Nostrand Company, 1927. Price, $4.50.

The Mathematics of Engineering, by Ralph E. Root, Ph.D. 550 pages, illustrations, 8vo. Baltimore, The Williams and Wilkins Company, 1927. Price, $7.50.

The Elements of Mining Science, by David E. Thomas. 80 pages, illustrations, 8vo. New York, Oxford University Press, 1927. Price, $1.50.

Ostwald's Klassiker der Exakten Wissenschaften No. 218: Abhandlungen über die hydrodynamische theorie der Schmiermittelreibung von N. Petrow, O. Reynolds, A. Sommerfeld und A. G. M. Michell. Herausgegeben von L. Hoff, Aachen. 227 pages, illustrations, 12mo. Leipzig, Akademische Verlagsgesellschaft m. b. H., 1927. Price, 9.60 marks.

A Short History of Physics, by H. Buckley, M.Sc. 263 pages, 12mo. London, Methuen and Company, 1927. Price, 7 shillings 6 pence.

Les Lampes a Plusieurs Électrodes et leurs applications en Radiotechnique, par J. Groszkowski. Traduit et adapté du Polonais par G. Teyssier. 348 pages, illustrations, 8vo. Paris, Étienne Chiron, 1927. Price, 40 francs.

Kinematics of Gerotors, by Myron F. Hill, A.B., A.M. Second edition. 44 pages, illustrations, 8vo. Philadelphia, The Peter Reilly Company, 1927.

Titanium, with special reference to the analysis of titaniferous substances, by William M. Thornton, Jr. 262 pages, illustrations, 8vo. New York, The Chemical Catalog Company, Inc., 1927. Price, $5.

General Chemistry, Theoretical and Descriptive, by Thomas P. McCutcheon, Ph.D., and Harry Seltz, Ph.D. 415 pages, 8vo. New York, D. Van Nostrand Company, 1927. Price, $3.50.

An Introduction to Organic Chemistry, by Roger J. Williams, Ph.D. 565 pages, 8vo. New York, D. Van Nostrand Company, 1927. Price, $3.50.

Elementary Physical Chemistry, adapted from a Treatise on Physical Chemistry, by Hugh S. Taylor, D.Sc. 531 pages, illustrations, plates, 8vo. New York, D. Van Nostrand Company, 1927. Price, $3.75.

The Romance of Chemistry, by William Foster, Ph.D. 468 pages, illustrations, plates, 8vo. New York, The Century Company, 1927. Price, $3.

Thermometric Conversion Chart for the Use of Mathematicians, Chemists, Physicists, Schools, Etc., by Percy L. Marks. 6 pages, colored plate. London, Crosby Lockwood and Sons, 1927. Price, 3 shillings 6 pence.

Abridged Scientific Publications from the Kodak Research Laboratories. Vol. x, 1926. 273 pages, illustrations, 8vo. Rochester, N. Y., Eastman Kodak Company, 1926.

U. S. Bureau of Mines: Bulletin 261, Resistance of Metal-mine Airways, by G. E. McElroy and A. S. Richardson. 149 pages, illustrations, folded plates, 8vo. Washington, Government Printing Office, 1927. Price, 35 cents.

Twenty-fourth Annual Report of the Bureau of Science, Philippine Islands, including summary of twenty-second and twenty-third annual reports to the Honorable, the Secretary of Agriculture and Natural Resources, by Wm. H. Brown, Director of the Bureau of Science for the year ending December 31, 1925. 61 pages, 8vo. Manila, Bureau of Printing, 1927.

CURRENT TOPICS.

An Investigation of the Rate of Growth of Crystals in Different Directions. MARIE BENTIVOGLIO. (*Proc. Roy. Soc.*, A770.)—This investigation was suggested by an article on the velocity of growth and dissolution of crystal faces published by G. Wulff in 1901. Miss Bentivoglio has been fortunate in devising a method of measuring the advance of a face perpendicular to itself that is more accurate than Wulff's and is of wider application. Besides this it makes it possible to use the same crystal again and again in a series of measurements. A hole was drilled in the seed crystal along an axis of a zone of faces to be investigated for growth. A pin was fixed in it and attached as a prolongation to a brass rod. This entire support could be placed in a definite and reproducible position on a horizontal plate of glass. By rotation of the brass rod a face was made horizontal. A microscope was sharply focussed on the face and the position of the adjusting screw read to .01 mm. After further growth the face was replaced exactly in the same position and a second reading taken. The difference of the two readings gave the growth. The tangential or lateral growth was not included in the investigation.

For the growing process the seed crystal was fixed in the centre of a spherical flask, containing 500 c.c. of solution and placed with its axis horizontal. By an electric motor the flask was rotated four or five times per hour. This assures equal opportunity of growth to all the faces in the zone under examination. A considerably higher rate of rotation made showers of small crystals to form while the mounted crystal grew irregularly.

The selection of working temperatures was an important factor in successful manipulation. "The initial temperature of saturation of the solution should not be too high on account of the difficulty in restraining the rate of cooling, and not too low in order that there may be a measurable amount of growth in a short time. The solution was put into the flask some 2° above its saturation point, to allow for initial cooling, and the crystal was thus introduced into a solution that was just about saturated. This precaution was necessary in order to guard against any initial dissolution of the crystal. On the other hand, the introduction of a crystal into an already supersaturated solution is to be avoided, as it certainly leads to irregular growth." Four or five hours was found to be a good duration for growth. Only those faces parallel to a common axis could be studied at the same time. Other zones of faces had to be made to grow in separate experiments. The range of measured growths was from .05 to 1.2 mm., but the growths usually lay between .3 and .8 mm.

Octahedra of potash alum and cubes of sodium chlorate were

the subjects of preliminary experiments, but the real investigation had to do with "isomorphous double sulphates of magnesium-ammonium, iron-ammonium and magnesium-potassium, of the general formula $R''R'_2(SO_4)_2.6H_2O$, belonging to the monoclinic system." In potash alum all the octahedral faces grew at the same rate. "It was observed that an initial irregularity, however marked, in the habit of the crystal had no influence on the subsequent rate of growth of its faces. Thus, some crystals were originally of regular habit, while others were flattened parallel to a pair of faces; but the equal growth of all faces clearly demonstrated that habit does not affect the rate of growth when no other form is present." Owing to the equal growth of faces an initially irregular or misshapen crystal tends to become more regular with the progress of growth, just as the disparity of five years in the ages of a man and his wife almost disappears by the time they are octogenarians. A different result comes when a rapidly growing face meets two slowly growing faces. The one of rapid growth forges ahead while the slow ones have to extend themselves laterally to meet it, so that they attain a larger area and the rapid face shrinks to a smaller and smaller area until it may almost disappear. In some salts examined one face grew two or three times as rapidly as another. Parallel faces by no means always grow at the same rate. Indeed, if a centre of symmetry is absent, the rates may be very different. G. F. S.

Electricity of Dust Clouds. G. B. DEODHAR, University of Allahabad. (*Proc. Phys. Soc.*, London, **39**, April 15, 1927.)—In tropical regions such as India, where large quantities of dust are blown into the air, a knowledge of the electricity developed thereby is of value because of its effect upon the insulation of laboratory apparatus and upon radio transmission. Not much has been found out concerning electricity caused by dust in motion. In 1913 W. A. D. Rudge published the results of investigation along this line, but much remains to be done. This Indian physicist sets himself to the problem with an energy almost incredible at the temperature of 40° C. at which the experiments were conducted.

A large cylindrical iron vessel has an opening in the bottom which is surrounded by an ebonite ring supporting wire gauze on which was placed the dust to be blown. There is a second orifice in the top of the vessel. The vessel is connected to a sensitive electrometer and is enclosed by an earth-connected wire gauze screen. Air is blown from beneath into the vessel and the dust is carried along. In this paper the electrical charge of the dust particles only was examined. In twenty cases the charge was positive and in nine cases negative. It seems impossible to discover a rule for predicting the sign of the charge. Iron filings become negative, while copper filings receive the opposite charge. Ebonite dust becomes negative and rubber dust positive. Mica dust sometimes is of one sign, sometimes of the other. Powders of some pure salts were

examined. NaCl and KCl raised the vessel to a potential of between one and two volts, but $NaNO_3$ developed a potential of 8.4 volts against 1.8 for KNO_3. A reduction in the size of the dust particles was followed by a great increase in potential, 46 volts being attained in one instance. "During a dust-storm a Geissler tube filled with helium was put in circuit with the aerial of our wireless set and the earth. It was found that the tube continued to glow quite brilliantly though intermittently. With the subsidence of the storm and settling of the dust the phenomenon vanished altogether, showing that during the storm the potential considerably exceeded the normal value." The author attributes the electricity of dust-storms to friction between particles of the dust. G. F. S.

Flame in the Wind. W. BURSTYN. (*Zeit. Technische Phys.*, No. 1, 1927.)—Fakirs often perform torch dances in which flames strike on their naked chests. In examining this the author found an interesting and analogous effect. A stream of air from a fan blows aside a flaming jet of gas issuing unthrottled from a pipe 15 or 20 mm. in diameter. The flame is spread out, loses its luminosity and makes a noise. The hand can be held without discomfort in this flame, provided the fingers are kept close together. A card may be substituted for the hand, but if there is a small hole in it its edges will burn. A current of cold air seems to be drawn in between the hand or card and the flame. G. F. S.

The Lightning Rod Has Reduced Farm Fire Losses. (U. S. Department of Agriculture Clip Sheet No. 472, July 17, 1927.)—Lightning, which annually causes large life and property losses, is, according to the U. S. Department of Agriculture, probably the chief cause of farm fires. Recently the seriousness of these losses caused the farm fire protection committee of the National Fire Protection Association to give special attention to this problem. Three of the members of this committee are from the Department of Agriculture, and the chairman is David J. Price of the Bureau of Chemistry and Soils of this department. This committee recently gave out the following statement regarding lightning-caused losses on the farm:

"The annual farm property loss from lightning may be estimated at $20,000,000 as a minimum. This loss can in large measure be prevented by proper lightning protection on buildings and by the simple expedient of grounding at reasonable intervals, all wire fences inclosing pastures or yards for livestock. The annual number of farm persons killed by lightning is between 400 and 500, and the number of such persons injured from this cause is more than twice the number of deaths.

"Available statistics indicate that lightning rods, both good and defective, as hitherto found on farm buildings have reduced lightning losses by about 85 per cent. of the loss incurred from lightning

on corresponding exposures of unrodded buildings, and that properly installed and well-maintained rods have shown an efficiency in the prevention of lightning damage of well nigh 100 per cent.

"A substantial metal roof with all parts thereof in good electric contact can, according to available evidence, be utilized as a part of the lightning-protection system for a building and thus in part be made to serve a double purpose. The cost of grounding and making other necessary electrical contacts with interior masses of metal is relatively small.

"The cost of lightning protection is more than justified when the added personal safety and avoidance of temporary deprivations, as well as the saving of property, are considered.

"The committee calls attention to a 'Safety code for protection of persons and buildings against lightning,' prepared by a committee working under the direction of the American Engineering Standards Committee. This committee consists of representatives of the United States Government and a number of national organizations. The code has been published in tentative form by the National Fire Protection Association, Boston, Mass."

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